BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles
Issue 51:
Magic Squares Redux

Share your problems, solutions, models, stories, and art:
https://aimathcircles.org/Bluebird

Even the seasons form a great circle in their changing, and always come back again to where they were. The life of a person is a circle from childhood to childhood, and so it is in everything where power moves.

—Black Elk, Oglala Lakota

NEWSFLASH Join LIVE Bluebird Math Circle to work on these activities together with friends and family.
Wednesday May 17, 5-6 PM MDT online.
Sign up at **https://aimathcircles.org/Bluebird**

MATH COYOTE CORNER
Question: Why should you worry about the math teacher holding graph paper?
Answer: She's definitely plotting something.

Warm-Up: Pairs of Squares

(a)

2	9	4
7	5	3
6	1	8

(b)

6	7	2
1	5	9
8	3	4

(c)

8	1	6
3	5	7
4	9	2

(d)

4	3	8
9	5	1
2	7	6

(e)

2	7	6
9	5	1
4	3	8

(f)

4	9	2
3	5	7
8	1	6

(g)

8	3	4
1	5	9
6	7	2

(h)

6	1	8
7	5	3
2	9	4

In Newsletter #49 we made 'magic squares'. These are 3x3 squares containing the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 arranged so that the sum of the numbers along each row, each column, and each diagonal is always the same.

On the left are eight such magic squares. Notice the following:

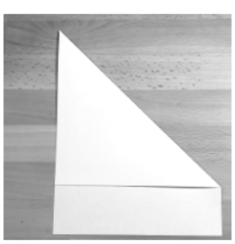
1. The magic sum is always 15.
2. The number 5 is always in the center square.
3. The even numbers are always in the corners.

There are many other things these eight squares have in common. For example, the numbers 2-9-4 always appear in a line (a row or a column). You never get 2-8-5 or 3-9-4.

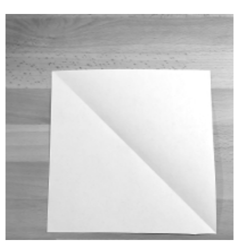
In fact, you can move each square to get to one of the others. For example, square (a) can be rotated by 90 degrees (clockwise) to get square (b). Pick two squares, and tell how each can be moved around to look like the other square. Then do the same thing for 3 other pairs of squares.

Family Circle: Magic Square Transformers

The eight magic squares in the warm-up are all *related* to one another: we call that *a family*. How can you make your own family of magic squares? One way: keep solving magic squares from scratch, as we did in Issue 49—and then, pair them up and check if any two are related. That's a lot of hard work! Instead, let's make a model that *transforms* one magic square to make its family.



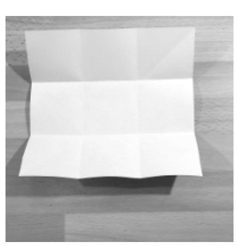
1. Take a sheet of plain paper. Fold the top edge of the sheet to the side edge.



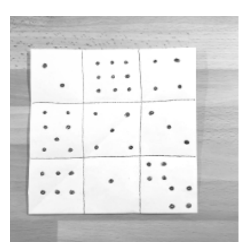
2. Cut or tear off the extra flap on the bottom. Unfold. It's a square (not yet magic).

3. Fold the top and the bottom thirds of your square over the middle third. This is hard! Wiggle the paper around until it lines up.

4. Crease the folds and unfold. Your crease lines split the square into three strips.



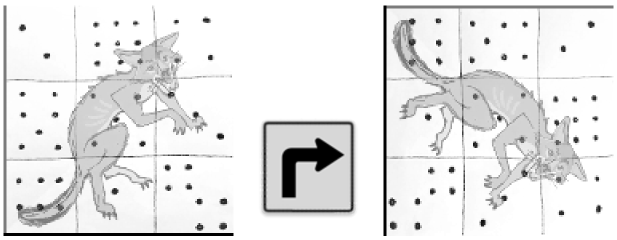
5. Repeat Step 3 and Step 4, but left to right. Your crease lines split the square into nine small squares. (Magic, next!)

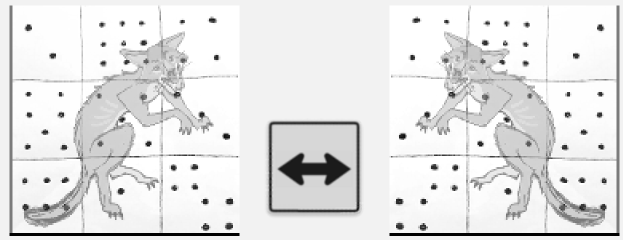


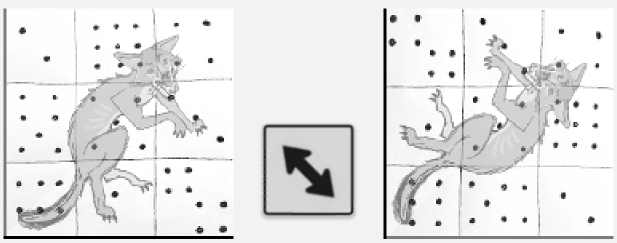
6. Find one solved magic square. We used (a) from the warm-up. Instead of numbers, draw as many dots in each small square.

Next, turn your sheet over and trace the dots on the other side, so the patterns are the same on the front and the back of the sheet. Modeling hints: Use your thumbnail to make paper creases sharper. Pick a dark color to see the dots through the paper easier. Arrange dots in patterns to count them at a glance. Take your dot patterns from dice or playing cards, or make up your own. It's totally fine if the model comes out a bit rough, as long as it helps you imagine the mathematics.

You are ready to transform your magic square and create its family! Here is our trickster Coyote showing examples of the transformations you can use: *90-degree rotations* and *line reflections*. Try these transformations on your own model.







Here are the magic squares Coyote's transformations create:

6	7	2
1	5	9
8	3	4

4	9	2
3	5	7
8	1	6

8	3	4
1	5	9
6	7	2

Each transformation has a hope of adding a new square to your growing family. How many different magic squares can you make this way?

Ask Bluebird

QUESTION—*What is your favorite way to encrypt a message? That means to translate it into a secret code?* From Nigel Wilson. **BLUEBIRD SAYS**—Cryptography (the study of codes) is a major subject of recent mathematical research. It has applications from national defense to financial systems to software security. We use its results, for example, whenever we use an ATM machine or sign into a website. One of the most interesting codes ever devised is the Navajo language itself. During World War II, Navajo soldiers were sent on airplanes to communicate so that the enemy could not understand.

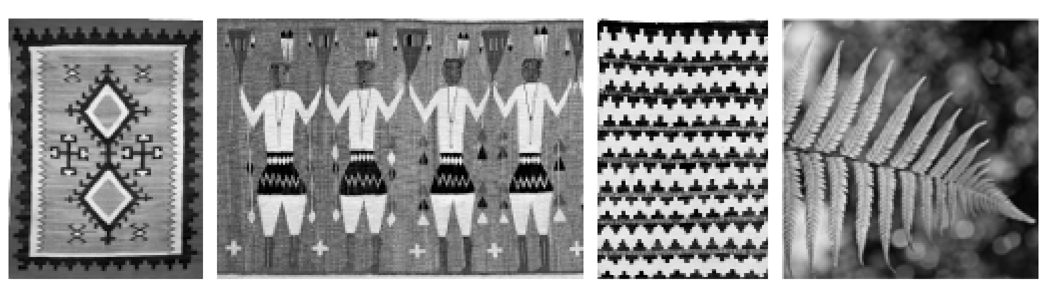
One of the easiest ways to encrypt a message is using substitution: Every time a particular letter occurs, you replace it with a particular symbol. The symbol may be a picture, a number, or even a letter. Here are some examples, using my own name:

'Plain text'	B	L	U	E	B	I	R	D	BLUEBIRD
Code 1	2	12	21	5	2	4	18	4	2 12 21 5 2 4 18 4
Code 2	C	M	V	F	C	J	S	E	CMVFCJSE
Code 3 ('cypher')	ɰ	ʃ	𝔛	ԁ	ɰ	ஐ	‼	ꓤ	ɰʃχ ԁ ɰஐ!! ꓤ

Notice that the two B's are always encoded with the same symbol. As it happens, my favorite code is easy to break, using ETAION SHRDLU. This is a list of letters in order of their frequency in English texts. You just count the occurrences of a particular symbol, and try to match it with one of the letters which has the same frequency. If you have a long enough text, this will work.

FUN FACT OF THE FORTNIGHT

Symmetry is an important mathematical concept. In its most general form, symmetry is a way of saying that one part of a figure or structure resembles (in some way) another part. Can you see how each of the following figures has some symmetry, in this sense?



Images: Navajo blanket, Honolulu Museum of Art; Yeibichai Lukachukai Navajo Rug; Navajo rug, Saint Louis Art Museum

Teacher Guide: https://aimathcircles.org/wp-content/uploads/2023/05/Bluebird-MC-Issue-51-Teacher-Guide.pdf