BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles
Issue 43
Sequences of Subsets: Binary Gray Code

Share your problems, solutions, models, stories, and art:
https://aimathcircles.org/Bluebird

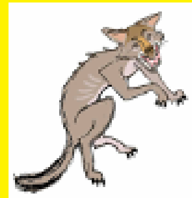
It does not require many words to speak the truth.

–Chief Joseph (Nez Perce)

NEWSFLASH Join LIVE Bluebird Math Circle to work on these activities together with friends and family.

Wednesday, January 25, 5:30 PM MST online

Sign up at **https://aimathcircles.org/Bluebird**



MATH COYOTE CORNER

I saw a bunch of mathematicians looking at some graph paper the other day. They must have been plotting something.

Warm Up: Read My Mind

SETS A set is just any collection of objects. Here are some examples of sets:





A set of drums, a set of kachina dolls, a set of cherries and a plate, a set of pots, and a set of various things.

The set of the first three letters of the alphabet.
The set of all the visible stars in the sky this evening.
The set of all numbers x such that 2x + 3 = 27.
The set of all the cats you can now see without moving from where you are now sitting.
The set of all positive integers.

The objects that belong to a set are called its *elements*. The set of the first three letters, of course, has three elements. The set of all positive integers has an infinite number of elements. The set of all the visible stars in the sky this evening has a certain number of elements for sure, but it would be hard for us to determine that number.
We can describe a set in different ways. Sometimes we describe it in words, giving a property that all its elements must have in common. That's what we've done above. But sometimes we don't want to be concerned, when we make a set, about how the objects in the set are related. Then we can use 'braces notation'. We just put braces around the elements of the set we are thinking above:

{hammer, scissors, screwdriver}
{hammer, scissors, screwdriver, watermelon}
{hammer, watermelon, 1, 2, 3}
{a, b, c, d, e}

Exercises: (you can do these in your head, or gather items in front of you to share with your friends!):
1. Make a set of three objects that you can now see.
2. Make a set of three objects that you cannot now see.
3. Make a set of three objects that have nothing to do with each other. Then ask someone else what these three objects have to do with each other! Ask a few people, then give a prize to the most imaginative answer.

SUBSETS Sometimes, one set is included in another: any element in the first set is also in the second set. Then the first set is called a *subset* of the second. If a set A is a subset of set B, we write $A \subset B$. (We can also write the same thing 'backwards': $B \supset A$.)

Suppose the set S = {a, b, c, d}. It has four elements. Can you list ALL its subsets?
Hints:
- There are 4 subsets with just three elements.
- There are 6 (yes, six!) subsets with just two elements.

It’s tricky, but there are also subsets with just one element: {a} ⊂ {a, b, c, d}. We sometimes call a set with just one element a *singleton* set. There are four singleton subsets of set S.

And there are two more subsets of S. One is S itself. Any set is a subset of itself: every element in the set S is “also” in the set S. The last subset of S is the trickiest of all. It is the set with NO elements: {} ⊂ S. It is strange but true that any element in {} is also an element of S. (Well, can you name an element of {} which is not in S?).

So how many subsets altogether does the set {a, b, c, d} have?

Exercises:

4. How many subsets does the set T = {pig, cow, dog, carrot} have?
5. How many subsets does the set U = {a, b, c, d, e} have? Hint: can you use what we know about the set S = {a, b, c, d} to answer this question?

Family Circle: Matching Subsets

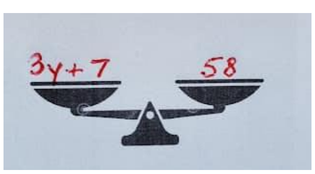
Here are two lists. Can you match each set in the first list with at least one set in the second list, so that the set in the first list is a subset of the set in the second list? There may be more than one way to make these matches. You can draw lines or arrows from the first list to the second to indicate the subset relation you’ve found.

THE FIRST LIST	THE SECOND LIST
{2, 4, 6}	{2, 4, 6, elephant}
{2, 4, elephant}	{2, 4, 6, teacup}
The set of all multiples of 4.	The set of positive numbers
The set of all numbers x such that x < x+1	The set of all numbers
{elephant}	The set of even numbers
{elephant, owl}	The set of words in English beginning with a T
{teacup, table, towel}	The set of all left shoes in the whole world.
The set of all right shoes in the whole world.	The set of all shoes in the whole world
The set of odd numbers	The set of all numbers
The set of all numbers	The set of words in English beginning with a vowel.

Ask Bluebird

***QUESTION**: How do you solve an algebraic equation with variables on both sides?*–from Ye-Shiao T.

BLUEBIRD SAYS: Interesting question. If an algebraic equation has ONE variable on ONE side, you can solve it by trial-and-error (a method often used in computer science). To solve the equation 3y+7 = 58, we can ‘plug in’ numbers for x:
Plug in y = 4 and we get 3x4 + 7 = 28 + 7 = 35. That’s too small. So try y= 20. We get 3x20 + 7 = 67. That’s too big. But it looks like the correct value is closer to 20 than to 4. So try y = 15: 3x15 + 7 = 52.



Only a bit too small. If we let y = 17, though, we get 3x17 + 7 = 58. Bingo!
But try this with an equation like 5y + 1 = 3y + 7. “Bigger” and “smaller” don’t work the same way. If we plug in y = 5, we get 3x5 + 7 = 22 on the right, and 5x5 +1 = 26 on the left. These two numbers are supposed to be equal. But should y be BIGGER than 5? Or smaller than 5? This method is not working.
So we can try to reduce the equation to one where all the variables are on the same side:
5y + 1 = 3y + 7
-3y   -3y
2y + 1 = 7.
And now we can proceed as before.

FUN FACT OF THE FORTNIGHT Usually, a set has more elements than its subsets–except for the set itself. But there are sets which are no bigger than their subsets: there are just as many natural numbers as there are even numbers(!). Look: 1↔2, 2↔4, 3↔6… and we never run out of either of them. Mathematicians call this a “one-to-one correspondence”. And any two sets for which there is a one-to-one correspondence must have the same number of elements.

Of course, this can only happen with infinite sets. In fact, one definition of an infinite set is a set that can be placed in a one-to-one correspondence with a proper subset of itself. You might read about fascinating properties of infinite sets in Issues and Recaps 16 and 17 https://aimathcircles.org/category/bbflyers/