BLUEBIRD MATH CIRCLE

Issue 44
Binary Search

Share your problems, solutions, models, stories, and art:
https://aimathcircles.org/Bluebird

There is but one secret to success: never give up.

—Ben Nighthorse Campbell (Cheyenne)

NEWSFLASH Join LIVE Bluebird Math Circle to work on these activities together with friends and family.

Wednesday, February 8, 12:30 PM MST online
Sign up at https://aimathcircles.org/Bluebird

MATH JOKE The farmer's daughter was good at math. She was helping her dad tend sheep. One day she told him: "I brought 30 sheep home from pasture."
The farmer replied: "But I only see 28. You used to be good at math!"
His daughter replied: "But I rounded them up!"

Warm Up: Sets and Subsets

NATIVE AMERICAN FLAGS Below are copies of the flags of eight Native American Nations.

Hopi Hualapai Isleta Pueblo Kalispel Kickapoo Laguna Pueblo Navajo Wichita (OK)

Source of images: https://www.tmealf.com/shop/native-american-flags

Two people play a guessing game. One of them writes the name of one of these eight nations on a secret piece of paper. The other person must guess which flag the first player has written down, by asking questions. But there are rules:

1. The question must have a 'yes' or 'no' answer.
2. The question must be answerable by looking at the flag or the name of the nation... You cannot ask, "Does this nation live in Colorado?" The reason is that the second player must be able to answer the question by just looking at the flag or naming the nation whose flag it is.

Play the game a few times. The object is to guess the flag in the smallest number of questions, every time.

If the player guessing is lucky, she will get the flag on the first try. For example, she might ask: "Is it the Navajo Flag?" She might get the answer 'yes', and guess with just one question. But if she is not lucky, she has only eliminated one of the eight flags. How might she eliminate more flags with each question?

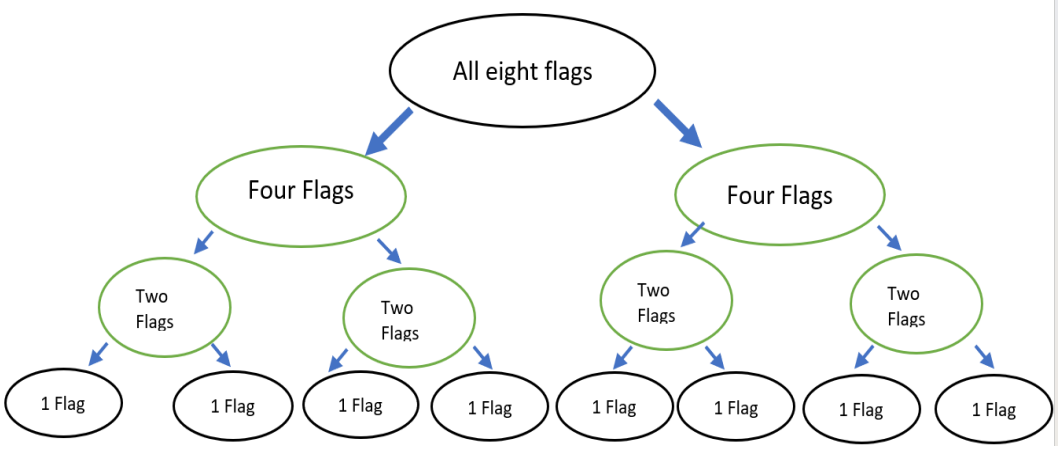
Remember: Do NOT assume that the player guessing is lucky. In fact, assume that he is UNlucky. In the worst case, what is the fewest number of questions with which he can win the game?

BINARY TREES One way to form a winning strategy is to keep cutting in half the number of flags that are still possible. For example, if you ask "Is your flag mostly blue", you will eliminate four flags. Can you see which ones?

Or, you could eliminate four flags by asking "Is the name of the nation below 'Kickapoo' in alphabetical order?"

If you keep halving the number of possible candidates, you are creating a *binary tree*.

For eight flags, you can narrow the guess down to a single flag with three questions.



The first question creates two sets of four, and tells which set the secret flag is in.
The second question creates four sets of two, and tells which set the secret flag is in.
The third question creates eight sets of one ("Singleton" sets). Once you have the singleton set, you can guess the flag.

Try to make a set of three questions for this set of flags which will 'catch' the secret flag in just three questions. Assume the worst luck for each question.

Family Circle

1. What if I were thinking of TWO flags, and you had to guess both? I will tell you if none of the flags have the property you ask about, or if one of them has that property, or if both of them have that property. That is, I can give you three answers to your questions. What would be a good way to guess?

2. Flags can get complicated. What would be a good set of 'yes/no' questions if we were guessing a number from 1 to 8? From 1 to 16? From 1 to 10?

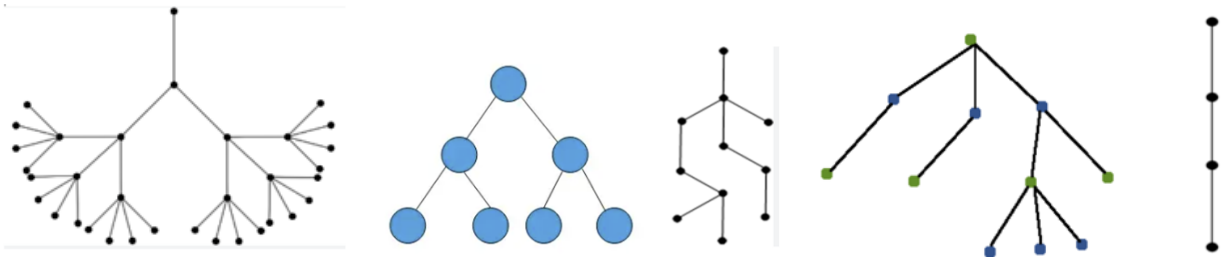
Teacher Guide: https://aimathcircles.org/wp-content/uploads/2023/02/Teacher-Guide-BMC-44-Binary-Search.pdf

Ask Bluebird

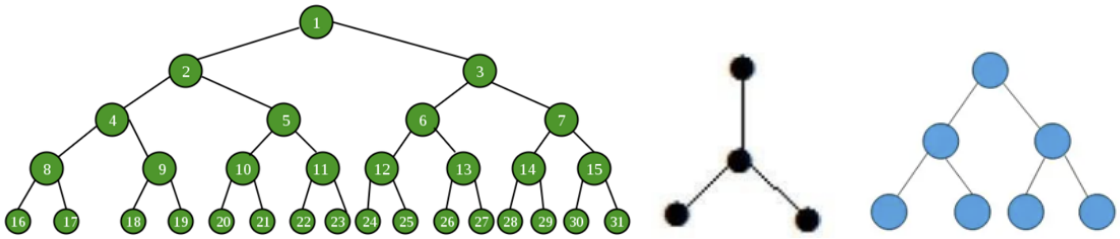
QUESTION—*What is bluebird's favorite shape?* From Wesley H.

BLUEBIRD SAYS—Interesting question. My favorite shape is a tree!

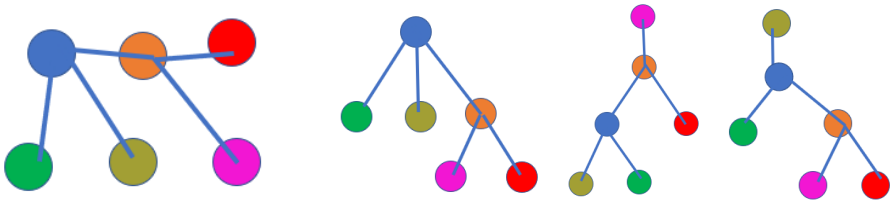
Is this a shape? Well, in mathematics it is! It is a set of points connected by line segments, but with no 'cycles'. You cannot go round and round in a tree. And mathematical trees usually grow down, not up. Some examples of trees:



My absolute favorite tree is a binary tree. Each node (branching point) has no more than two twigs (edges) coming out of it. Some examples of binary trees:



FUN FACT OF THE FORTNIGHT Any network ('graph') which has no loops or circuits is a tree! Just 'pick up' the network from one vertex and shake it out! This can be done in many ways with any particular graph without loops or circuits. The graph on the left can be drawn as a tree in several ways:



All that matters is that corresponding vertices (here, with the same colors) are connected in the same way.