



# BLUEBIRD MATH CIRCLE

## Issue 42: Stepping Stones and Brick Walls

Share your problems, solutions, models, stories, and art:  
<https://aimathcircles.org/Bluebird>

*Humankind has not woven the web of life. We are but one thread within it. Whatever we do to the web, we do to ourselves. All things are bound together. All things connect.*

—Chief Seattle, Duwamish  
(1780-1866)

Join LIVE Bluebird Math Circle to work on these activities together with friends and family.

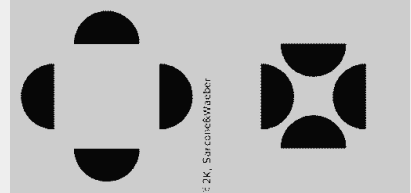
### NEWSFLASH

Wednesday, January 11, 12:30-1:30 PM MST online.

Sign up at <https://aimathcircles.org/Bluebird>

Compare the two images below (composed of 4 half discs). What do they have in common?

### MATH PUZZLE



## Inspiration: Wildflowers



Segoe lily



Scarlet globemallow



White mountain avens



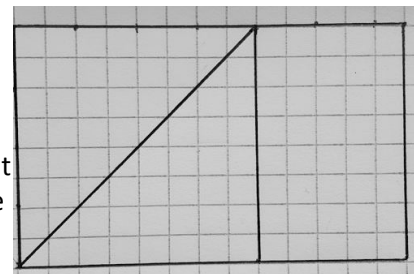
Common blanketflower

Three photographs were taken on the Navajo Nation and one in Colorado nearby. Can you tell which one?

## Warm-Up: Rectangles, Squares, Spirals

We start by drawing a 13-by-8 rectangle (can you guess where the dimensions come from?).

1. Divide the rectangle into the largest possible square and the remaining rectangle. What are its dimensions?
2. Draw a diagonal of the square (as in the picture)
3. Using the rectangle obtained at step 1, repeat steps 1 and 2. Draw your square so that one of its diagonals shares a point (at a corner of the square) with the diagonal of the square in step 2.
4. Repeat the process for as long as you can.
5. So far, we've been going **inside** the 13-by-8 rectangle. Can you think of a similar process but going **outside**?

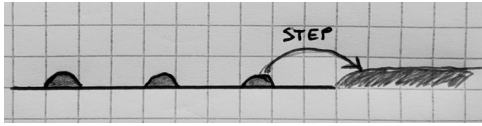


Let's list the side lengths of the squares which we obtained (in the increasing order) starting with the smallest ones:

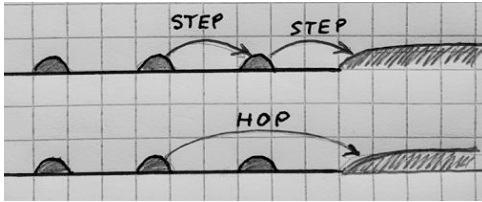
These numbers are called the Fibonacci numbers, and the sequence is called the Fibonacci sequence.

# Family Circle: Stepping Stones and Brick Walls

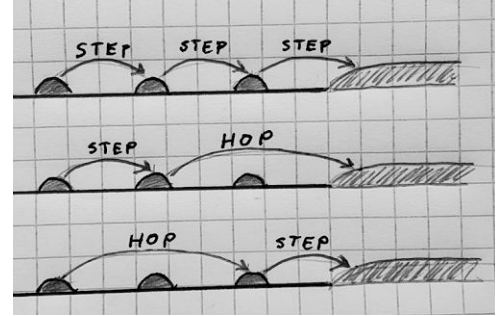
**Stepping Stones** A set of stepping stones allows you to cross a small river. How many ways to the bank are there if you are standing on one of these stones? You can either step on to the next stone or else hop over one stone to land on the next-but-one. If you are on stone number 1, you can only step (s) on to the bank: 1 route.



If you are on stone 2, you can either step on to stone 1 and then the bank (step, step or ss) OR you can hop directly onto the bank (h):



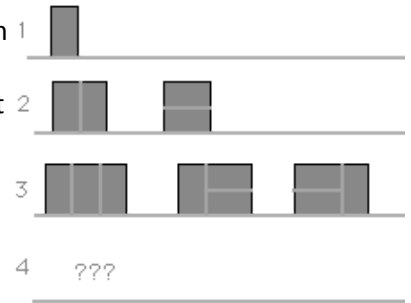
From stone 3, you can step, step, step (sss) or else hop over stone 2 and then step (hs) or else step on to stone 2 and then hop over stone 1 to the bank (sh):



How many ways do you have to cross the river if you are on stone 4? on stone 5? on stone 10?

**Brick wall patterns** The standard brick has a length twice as long as its height - its length is two units and its height is one unit. We want to build a low brick wall so that the wall is just two units tall. We can make our wall in a number of patterns, depending on how long we want it:

- There's just one wall pattern which is 1 unit long - made by putting the brick on its end.
- There are 2 patterns for a wall of length 2: two side-ways bricks laid on top of each other and two bricks long-ways up put next to each other.
- There are three patterns for walls of length 3.
- How many patterns can you find for a wall of length 4? of length 5? Of length 10?



## Ask Bluebird



**QUESTION**—How does a cuckoo clock work? - from Mythili G.

**BLUEBIRD SAYS**—The classic cuckoo clock is composed of three essential parts — the pendulum, “weights,” and the chains that you pull to “wind up” the clock. It’s the movement of these components, along with gravity that makes the hands move and the clock work. As the day progresses, the weights slowly drop to their lowest position while the pendulum swings at a steady and even beat. The speed at which the pendulum moves back and forth determines how the hands move on the clock. The weights serve two purposes. One drives the time-keeping element of the clock, while the second one controls the cuckoo call and the bird’s mechanics. The weights help the



in and out movements of the bird. A bellow and whistle work together to make the “cuckoo” sound. The chains connect to the chain gear that works the internal operations. You can read more about cuckoo clocks in a 2013 article in Smithsonian Magazine:

<https://www.smithsonianmag.com/arts-culture/the-past-present-and-future-of-the-cuckoo-clock-65073025/>

Photo: An early Black Forest cuckoo clock, circa 1870 at Deutsches Uhrenmuseum, Furtwangen (Germany)

**FUN FACT OF THE FORTNIGHT** There are only four 4-digit Fibonacci numbers - 1597, 2584, 4181 and 6765 - so 2023 isn't one. But we can observe a remarkable property of 2023:

$$2023 = (2 + 0 + 2 + 3) \cdot (2^2 + 0^2 + 2^2 + 3^2)^2 = 7 \cdot 17^2$$

So its smallest prime factor 7 is the sum of its digits, and its largest prime factor 17 is the sum of the squares of its digits! And 2023 is the only 4-digit number with this property. In fact, numbers with this property are pretty scarce - here are the first six of the sequence of such numbers: 133, 803, **2023**, 106811, 383177, 1071949. This is sequence [A217690](https://oeis.org/A217690) of the On-line Encyclopedia of Integer Sequences (OEIS) <https://oeis.org/>

0 1 3 6 2 7  
: 13  
: OE 20  
23 IS 12  
10 22 11 21