BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles
Issue 35
Binary Gray Code

Share your problems, solutions, models, stories, and art: **https://aimathcircles.org/Bluebird**

As indigenous people, way before the United States, or Canada or European invasions, we had our own knowledge systems. A lot of them were very scientific in terms of how they related to issues of the environment, sustainability, resilience and climate change.

—Shanondora Billiot, United Houma Nation
Quoted in TheHill.com

NEWSFLASH Join LIVE Bluebird Math Circle to work on these activities together with friends and family.
Wednesday, September 21, 5-6 PM MDT online.

Sign up at **https://aimathcircles.org/Bluebird**

MATH JOKE

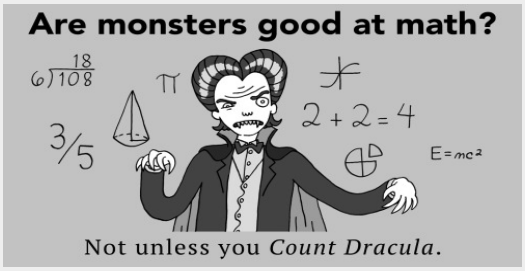


Image credit: spring-of-mathematics.tumblr.com

NOTE: This is Part II of the Binary Gray Code explorations. For Part I, see Bluebird Newsletter 34 at https://aimathcircles.org/Bluebird

Warm Up: Ordered Triples

Inspiration: There are three lightbulbs on a string, and each one can be on or off. Can you list all possible states of the string?

Image credits: Garfield, Westmenlights.

An ordered triple of numbers is just a list of three numbers in a particular order. In this activity, each of the three numbers will be a 0 or a 1. But in more general work, each could be any number at all. The three numbers are called *coordinates*.

If you are familiar with the notion of coordinates in geometry, ordered triples are just the coordinates of a point in 3-space.

For example, (0,1,1) is an ordered triple. It is different from the ordered triple (1,0,1). In both cases, there are two coordinates 1 and one coordinate 0. But they are in different places.

We have just used the standard notation for an ordered triple: three numbers separated by commas and enclosed in parentheses. But for short, in this activity, we will write '011' for the ordered triple (0,1,1), and so on. We hope this causes no confusion.

It is not hard to see that there are eight ordered triples in which each number is either 0 or 1. Here is a list of all eight:

100
101
111
110
010
011
001
000

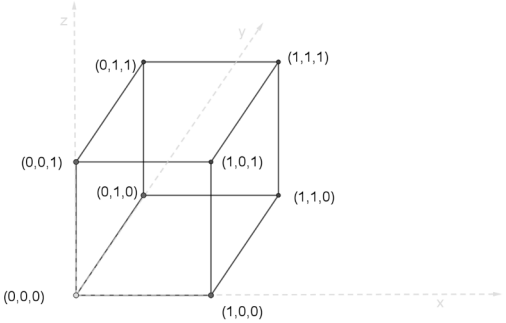
This list has the property that going from each ordered triple to the next, we change exactly one coordinate, either from 0 to 1 or from 1 to 0.

Activity 1: Find two more such lists.

Activity 2: the list above has the interesting property that it 'loops': the last ordered triple 000 can be changed to the first 100 by changing exactly one coordinate (the first coordinate). Can you find two more lists of our eight ordered triples with this property?

Family Circle: A Geometric Interpretation

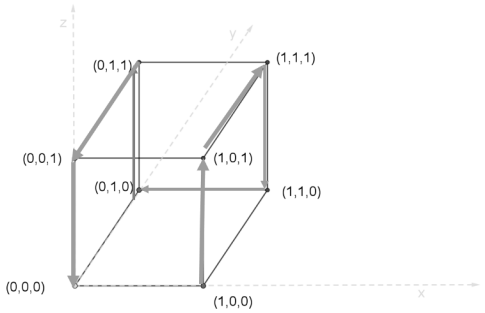
It turns out that the task we set ourselves above has a geometric interpretation. We take a unit cube, and set up a coordinate system so that the cube's vertices are exactly the ordered triples we are discussing. The result is on the right.



Then each of our lists corresponds to a path around the cube which passes through each vertex exactly once. For example, the list

100
101
111
110
010
011
001
000

corresponds to the path on the right.



Activity 3: Take your two lists from activity 1 and draw the corresponding pathway on the cube. How are the pathways the same? How do they differ?

Ask Bluebird

QUESTION: *Where do numbers come from?*—from Mae A.

BLUEBIRD SAYS: Great question! And one that can be interpreted in many ways. The short answer: We create numbers. They come from our minds. Archaeologists have found 'scorings' on cave walls, markings that show that someone has been counting something. And that something was probably days or months. Keeping a calendar was central to the survival of early peoples, in regions where the seasons change. Astronomy requires arithmetic, and sophisticated calendars, such as the Egyptian, Jewish, or Maya calendars, required sophisticated mathematics—much more than just numbers. So the development of numbers began very early.

But some cultures don't seem to have developed numbers, or not many numbers. Anthropologists have worked with people who don't have words for numbers greater than three or four. It seems that the development of numbers happened when the numbers were needed.

"Numbers" could also refer to our written symbols for numbers. These are much more recent. The particular symbols we use originated in the middle east and came to Europe, then to America, in early modern times.

FUN FACT OF THE FORTNIGHT A symmetry of a figure is a way to move it onto itself so it looks as if it hasn't been moved at all. For example, if we 'flip' an isosceles triangle around the altitude to its base, the various points in the figure move around, but the figure as a whole stays the same.

It turns out that if you look at the lists we have been considering, and add the requirement that they form a loop, so that the last ordered triple leads back to the first (as the given list does), then all the corresponding pathways around the cube look the same. They can be made to coincide by one of the various *symmetries* of the cube. There are 48 symmetries of the cube.