BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles

Issue 33 Recap
Three Old and One New Problem

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NEWSFLASH Join LIVE Bluebird Math Circle with friends and family.

September 7th, 5-6 PM MDT online.

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Introduction

After a peaceful welcome with the song of a bluebird, Donna Fernandez introduced our math circle.

The newsletter Issue 33 started with telling people that Bluebird also had a vacation and was able to do some traveling. One of the places that Bluebird got to go to was Sonoma County in California where he met with a number of students and other community members from the Pomo tribe. From the picture in the newsletter you can see that students created some great 3d shapes using Zome tools. And they also learned about our math circles.

Bluebird also got to see some of the beautiful dancing - the Shake Head Dance of the Pomo tribe.

This is a social dance that Pomo people hold in the spring and in the summer, so it's a chance for people to visit other family members from other tribes at this social event.

And then Bluebird also got to go to Zuni, New Mexico, and met with members of the Zuni tribe out there. Some pictures from Zuni:

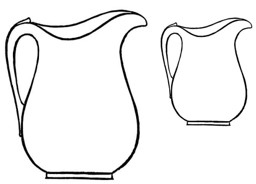
And so it was really a good summer break for Bluebird!

And then Donna asked people, " What was one thing that was memorable about your summer?"
Here are some of the answers:
– *One thing that was really memorable from my summer was getting my guitar* - Mya
– *I completed level books* - Damian
– *I remember about my summer just hanging out with people* - Rena
– *One memorable thing about my summer was going to Vegas* - Mikayla
– *I mostly read* - Daniela
– *One thing I did in the summer was that I went to my grandma's house in Albuquerque* - Darius
– *Something memorable about the summer is going back to school* - Melvtin

Wow, this last one is always good to hear as a teacher!

At this point we were eager to start solving problems.

Family Circle: Three Old Problems

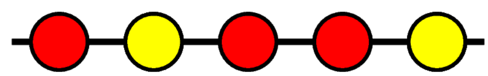


Pitcher Perfect!
You are visiting your grandmother and notice that she has two pitchers.

Problem 1: Your grandmother remembers that one of the pitchers holds 5 cups of water, and the second holds either 3 or 4 cups, but she's not sure which. By just using the two pitchers, how could you determine how much the second pitcher holds?

Problem 2: Suppose instead that one of the pitchers holds 5 cups of water and the other holds 12 cups. Your grandmother asks you to measure one cup of water for her. Can you do that? Could you measure one cup of water if instead one pitcher holds 4 cups and the other holds 12 cups?

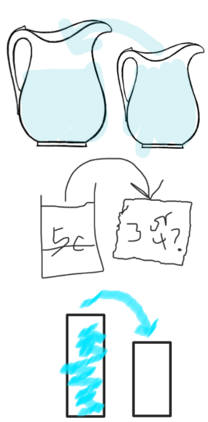
Problem 3: Crafty Math
You find some red and yellow beads around your house and decide to make a gift for each of your friends by stringing the beads together into a necklace. You have more red beads than yellow so you decide to make each necklace with three red beads and two yellow. You also want each of your friends to have their own special necklace that is not like any of the others. How many different necklaces can you make like this?



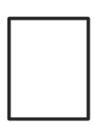
Oops. With three red and two yellow beads, you can't make enough different necklaces for all of your friends. What if you make them instead with four red and two yellow? Or five red and two yellow?

Each participant chose one of these three problems, and then we split into groups and went to separate breakout rooms to solve our problem of choice.

When we came back, it turned out that we had defeated each problem! Below are solutions. Here's how our participants kept notes of their mathematical thoughts as they worked on the solutions:



5 poured into the smaller pitcher and poured the rest into a measuring cup.

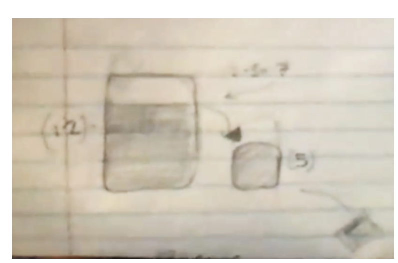


fill the big one, fill the second one from the big one. pour the second one out and keep the first one somewhere else. Do it again. If you can pour both the remnants into the smaller jug, it is 4 cups.

5 - 4 = 1
save
5 - 4 = 1

1 + 1

5 - 3 = 2 save
5 - 3 = 2 save
2 + 2 pour back in and if it over flows then it must be 3 cups



Participants in one of the groups solving a water-pouring puzzle wanted to have a reservoir to save all their water, so it's not wasted down the drain. Some wanted to imagine an infinite reservoir (abstract) and some wanted a realistic large bathtub (200 cups, they said). At the end, they decided that choice doesn't matter for the math in the problem, but might help people focus. It's better for people solving the problem to imagine whatever sparks their joy.

For both problems 1 and 2 it will be convenient to introduce a good way of recording our actions. One possible is described here:

> If we have a large 10-cup pitcher and a small 7-cup pitcher, the chain 0,0 → 10,0 → 3,7 → 3,0 has the following meaning: we start with two empty pithers; fill in the large pitcher; pour from large pitcher into the small pitcher (filling it); then dump out the water from the small pitcher. At this point, people got unhappy - we don't want to waste water! Thus we've agreed that we dump water into a large tup, thus preserving it for some future use. Another possibility would be to water plants with this water we dump from a pitcher.
>
> One more convenient agreement is to call a pair of numbers which describe how much water does each pitcher contain (like 0,0 or 10,0 or 3,7 or 3,0) a 'state' and every arrow, an 'action' (the first arrow in the chain indicates filling in the large pitcher; the second arrow indicates pouring 7 cups from the large pitcher into the small pitcher; the third arrow corresponds to dumping 7 cups from the small pitcher into the tub.

With this notation we are ready to describe our solutions for problems 1 and 2.

Problem 1 solution.
If the small pitcher holds 3 cups, we could have the following chain:
0,0 → 0,3 → 3,0 → 3,3 → 5,1 → 0,1 → 1,0 → 1,3 → 4,0

We fill up the small pitcher, pour from the small to the large, fill up the small one again, pour from it into the large one; dump water from the large one into the tub, pour from the small into the large pitcher, fill up the small one, pour from the small one into the large one. After performing these eight actions (count the arrows in the chain!) we end up with the large pitcher 'almost' full and the small pitcher empty.

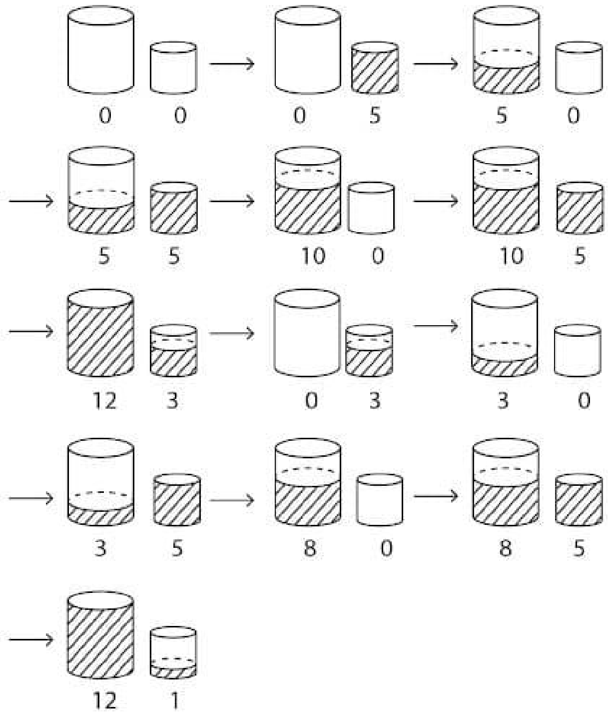
Let's now check what would happen if the small pitcher holds 4 cups and we perform the same eight actions:
0,0 → 0,4 → 4,0 → 4,4 → 5,3 → 0,3 → 3,0 → 3,4 → 5,2

We end up with the large pitcher completely full, and the small one non-empty!

Hence all we need to do is perform these eight actions in the order described. If we end up with an empty small pitcher it means that it holds 3 cups. If we end up with the small pitcher having water then it means that it holds 4 cups.

Problem 2 solution.
The following chain provides one solution:
0,0 → 0,5 → 5,0 → 5,5 → 10,0 → 10,5 → 12,3 → 0,3 → 3,0 → 3,5 → 8,0 → 8,5 → 12, 1.

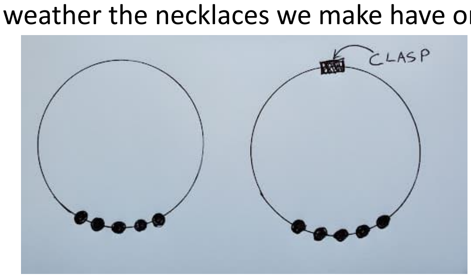
It shows that we can obtain exactly 1 cup in the small pitcher after performing 12 actions. The picture below illustrates this process.



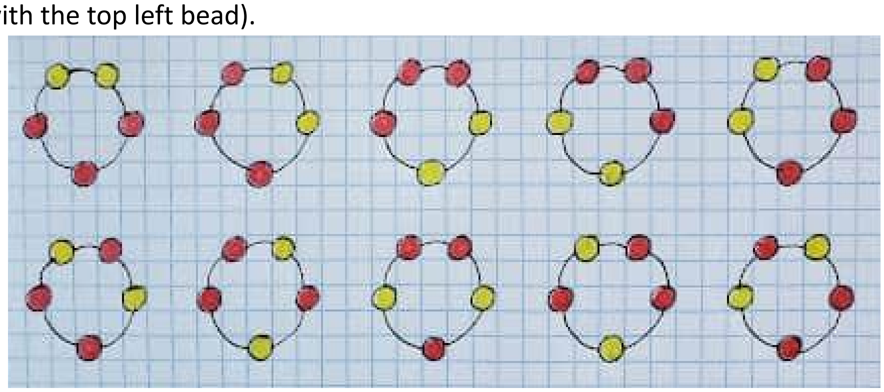
Try to find other solutions. See if you can find a way to obtain exactly one cup using fewer steps.

Problem 3 solution.
To begin with, we can see that altogether there are 10 ways to arrange 2 yellow and 3 red beads in a row:
YYRRR, YRYRR, YRRYR, YRRRY, RYYRR, RYRYR, RYRRY, RRYYR, RRYRY, RRRYY

But if we imagine these beads on a necklace we can see that some of the arrangements produce the same necklaces. And the number of different necklaces depends on weather the necklaces we make have or don't have a clasp:



If there is no clasp, we can rotate beads around the entire necklace, and each rotation will produce a different linear arrangement, but it's still the same necklace. In the picture below we exhibit all ten linear arrangements (for example, read them clockwise starting with the top left bead).



We can see that the five arrangements in the top row are all obtained from just one necklace. Likewise, the bottom row shows five remaining arrangements, and again, they are all obtained from just one necklace. Therefore we conclude that if we make necklaces with no clasp then we can only make 2 different necklaces.

If the necklaces have a clasp, the situation is different. The beads can't be rotated around the necklace. But we can flip the entire necklace around! And after some checking we see that there are 6 different necklaces with a clasp. We can list them starting with the leftmost bead:
YYRRR (it's the same as RRRYY when flipped)
YRYRR (the same as RRYRY)
YRRRY
RYYRR (the same as RRYYR)
YRRYR (the same as RYRRY)
RYRYR

We didn't have time to study the cases with 4 red and 2 yellow beads, or with 5 red and 2 yellow beads. We would love to hear from you if you investigate these cases. Please, share your findings with Bluebird.

More Mathematics for a Curious Reader

Pitcher problems can be used as an introduction to at least two deep mathematical ideas - the *Euclidean Algorithm* and *Diophantine Equations*. Besides a lot of beautiful mathematics, these ideas have many applications. For example, they help with cybersecurity: coding and decoding data to keep it private.

The Euclidean Algorithm is used to find the Greatest Common Divisor (GCD; school textbooks often use the term Greatest Common Factor (GCF) in place of GCD. Mathematicians prefer the term GCD). A more familiar way which uses representing numbers as products of prime powers isn't practical for large numbers while the Euclidean Algorithm remains surprisingly efficient. It is based on a very simple idea of dividing with remainders.

Besides being a practical way of finding the GCD of large numbers, the Euclidean Algorithm plays a very important role in establishing many number-theoretical results. In particular, it can be used to show that the GCD of two numbers can be written as their linear combination.

A Diophantine Equation is an equation for which we seek only integer solutions. Probably the most famous one is the equation $x^n + y^n = z^n$. For nearly four hundred years this equation had defied many generations of first-class mathematicians; only a few years ago it was proved that such an equation does not have solutions in positive integers if n is larger than 2. Simplest Diophantine equations are the ones known as linear equations - they are of the form $ax + by = c$, where a, b, and c are known and one needs to find suitable integer values of x and y. Most pitcher problems are some variations of this general idea.

Despite the relative ease with which Diophantine Equations can be stated, there are many deep and even unsolved problems in this area of mathematics.

Necklace Problems.
These problems provide an excellent opportunity to take a peek into an area of mathematics called *Group Theory*. Although we solved the problems in this issue with not much trouble, just imagine a situation when we need to deal with a similar problem about necklaces containing thousands beads of hundreds of different colors. The task will become quite daunting! Fortunately, group theory provides tools for solving problems of this kind. In particular, a theorem known as Burnside's Theorem converts it into a pleasant exercise. We hope that some of our readers will have the pleasure of learning group theory and its many wonders.

Share your ideas with other Bluebird Math Circle participants at https://aimathcircles.org/Bluebird

New Questions for Bluebird

What is a recursive formula? - From Donna Fernandez

What is the minimum of given numbers that you must have to solve Sudoku? - From Daniela U.

Is a formula the same as an equation? - From Tatiana Shubin

BLUEBIRD SAYS—Fascinating questions! I will fly around and seek answers. Watch this space in the next newsletter issues!

Submit your math-related questions at https://aimathcircles.org/Bluebird