BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles

Issue 34 Recap
Sequences of Subsets: Binary Gray Code

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Introduction

Donna Fernandez introduced our math circle. Before we tell you all about it, here are our tools and supplies:

- Newsletter: https://aimathcircles.org/issue-34-sequences-of-subsets-full-color-pdf/
- Jamboard (including participants' work): https://jamboard.google.com/d/1XNNsS_D9VbUWh-cc4brvE7EE_5GtpgFADDsWJzlwS4A/viewer?f=0

Sequences of Subsets

We first reviewed the notions of a set and its subsets. The following fine points were covered:

Some sets are 'singleton' sets, with just one element: {elephant}, {2}, {scissors}.
The null set is defined as the set with no elements. We can write it as {} or with the special symbol Ø.
The null set 'counts' as a subset of any set.
Any set 'counts' as a subset of itself.

We then looked at the set {a,b} consisting of two elements. We found that it has four subsets:

Ø, {a}, {b}, {a,b}.

(We started writing {ab} to mean the set {a,b}. It didn't cause confusion. We hope it doesn't cause confusion in this recap either.)

Then we set ourselves the task of listing these four subsets in a particular order. The rule we set ourselves was:
To get from each subset to the next subset, we are allowed only to insert one new element or delete one old element. Call these 'good' sequences of subsets.

Here are some good sequences:

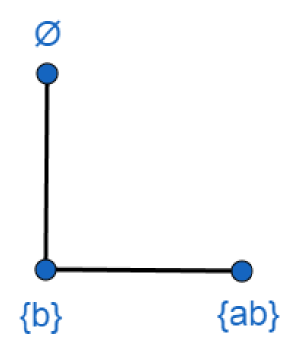
{a}, {ab}, {b}, Ø.
Ø, {b}, {ab}, {a}.

Here is a bad ('not good') sequence:

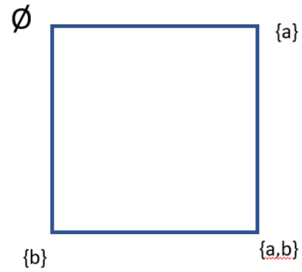
{a}, {b}, {ab}, Ø.

The problem with this sequence is that going from {a} to {b}, we must do two things: delete 'a' and insert 'b'. We are allowed only one of these. The same problem occurs in this sequence going from {ab} to Ø: we must delete 'a' and also delete 'b'.

Working together, we found eight such sequences. In order to determine if we had them all, we sought to put them in some order. Starting, say, with {b}, we could get to two other subsets: {ab} or Ø. So we drew a 'map', or graph, of how these subsets are related.

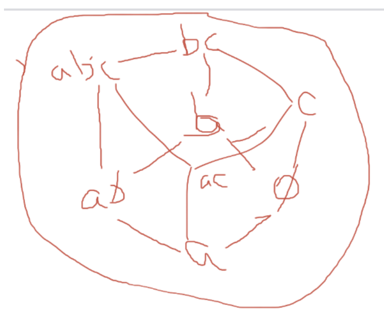


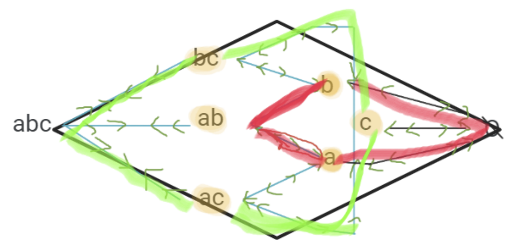
Continuing this process, we got to the graph below, which is just a square with each vertex labeled with one of the subsets:



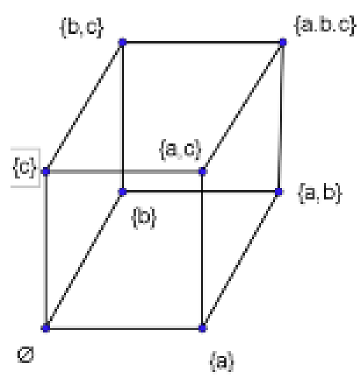
From any vertex, we can get to any other 'legally' by following a side of the square. There are four choices of initial vertex, and two choices of directions to go in, so there are eight possible good sequences.

Our next challenge was to do the same thing with the set {abc}. This set has eight subsets, and we got a few 'good' sequences (using the same rules). We drew diagrams showing the sequences:





(These were among a number of diagrams we drew.) We worked on these diagrams, to get something like this, which is analogous to the situation where we considered subsets of the set {a,b}:



At this point, we ran out of time! But many fertile speculations were advanced:

- What if we drew other diagrams for these subsets? Would they be topologically equivalent to the cube we drew?
- In two dimensions (with subsets of {a,b}), every good sequence was a 'loop': the last element connected with the first. Is this always true about subsets of {a,b,c}?
- Can we count the good sequences of {a,b,c}? (This question connects with counting the symmetries of a cube, which is not a simple task.)
- What if we looked at subsets of {a,b,c,d}? Would we have a geometric model for these? (It turns out that the natural geometric model is a four-dimensional(!) cube.)

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