BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles

Issue 31 Recap: Stitch It

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Monday, July 25, 5-6 PM MDT online.

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Introduction

After a peaceful welcome with the song of a bluebird, we began by posing a couple of new questions to Bluebird.

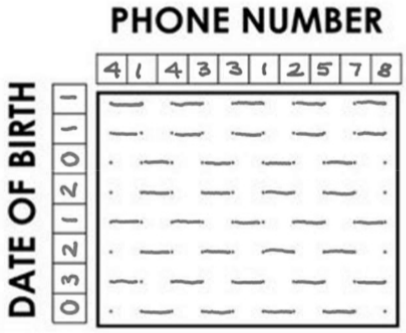
Stitch It

Sashiko is a style of Japanese embroidery used to repair, mend, strengthen and decorate fabric. It is made with running stitches that create geometric patterns.

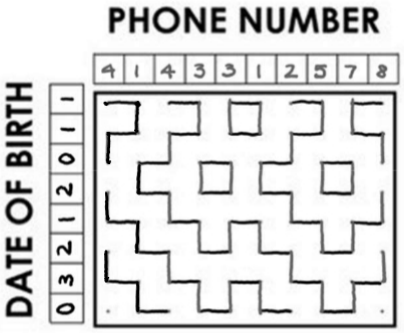
Images by upcyclestitches.com, zenstitching.ca, bluelela, and susanbriscoe.blogspot.com

Hitomezashi is a special style of sashiko where the pattern comes from the interplay of horizontal and vertical stitches on a grid. I first learned about hitomezashi from the blog #MathArtChallenge (Day 14) https://arbitrarilyclose.com/home/ which contains many great activities connecting math and art.

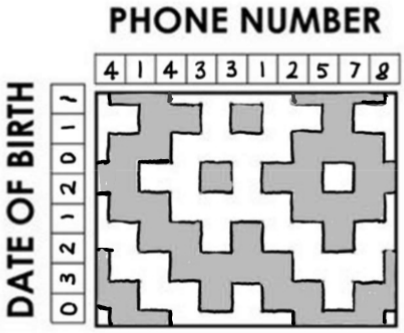
A good first activity to introduce the rules of hitomezashi was suggested by Sarah Carter on Twitter.



Horizontal stitches: odd number: start on the first dot, connect to second;
even number: start on second dot, connect to third



Vertical stitches: do the same with the phone number guiding your vertical stitches



You can color the pattern - neighboring regions must have a different color

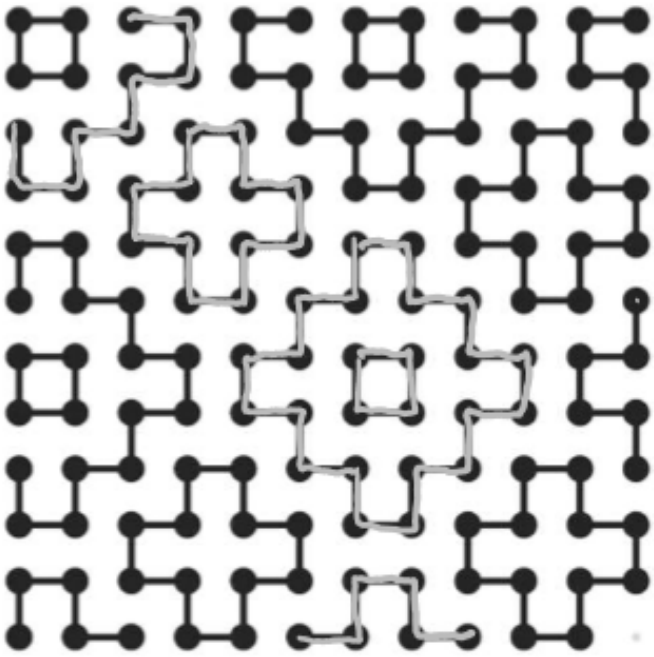
It is important to note that in every row and column we make only one decision: start at the first dot, or the second. From that point on, all the stitches are determined, since they alternate being above and below the fabric. It is important to note that each stitch has the same (unit) length. Also, we need to continue the stitches to the boundary to be able to see the regions for coloring. This means that when we started on the second dot which is connected to the third there would be a stitch before that connecting the '0th dot' to the first. Only a small piece of this stitch is inside the boundary but it is necessary to see the regions for coloring. The rightmost picture shows these stitches that connect to the boundary both at the beginning and end of certain rows and columns.

We discussed how to make this small pattern and several examples were produced. Danielle showed her work, as did other participants. In the patterns made some of us used a repeating string of 0s and 1s, others used a random sequence. Nigel pointed out that it would be interesting to see how the resulting patterns are different.

Maria remarked that it is very satisfying to see how the pattern emerges and it made her wonder what happened to the pattern if we changed our repeating string from 001100… to 0011100… or 00111100… and so on, or instead try 00100110011100111100… with an increasing number of 1s in between two 0s. These ideas reminded Nigel of the multiplication of numbers containing 1s, that is, 11x11=121, 111x111=12321, etc. While this pattern breaks down after the middle number reaches 9, it is a great example for students to see. (An entertaining description of this and other curious patterns can be found in The Number Devil by Hans Magnus Enzensberger.)

Nigel brought up the question whether every pattern could always be colored by two colors in such a way that neighboring regions have a different color. This is indeed the case - we returned to this question later in the session. He also thought it would be useful to have a tool that can generate and color patterns fast for more exploration. There are several hitomezashi pattern generators online. One particularly nice one is https://fernandojerez.com/hitomezashi/ while https://hitomezashi.com/ starts with the traditional Japanese indigo colored background and white thread.

Next, we talked about making bigger patterns on a grid paper or using online resources, in particular, Mathigon's Polypad https://mathigon.org/polypad . Polypad is a great tool to quickly make fairly large patterns, although coloring the regions is not very easy. It can be done using a pencil tool just like on paper though. We can use words and expressions to generate the hitomezashi pattern. For example, one can code consonants as 1 and vowels as 0. Maria explored a different way of coding letters, so that the actual letters could be recovered from the pattern. For example, a binary coding of each letter's position in the alphabet might work for this purpose, and one could think about how to use hitomezashi to send secret messages. However, one would need to think about how the different potential blocks representing different letters can be put together to form a regular pattern. This idea is connected to tessellations and tiling and we talked about the conditions that a certain rectangular pattern could be repeated vertically and horizontally so that it is guaranteed that the resulting pattern obeys the hitomezashi rules.



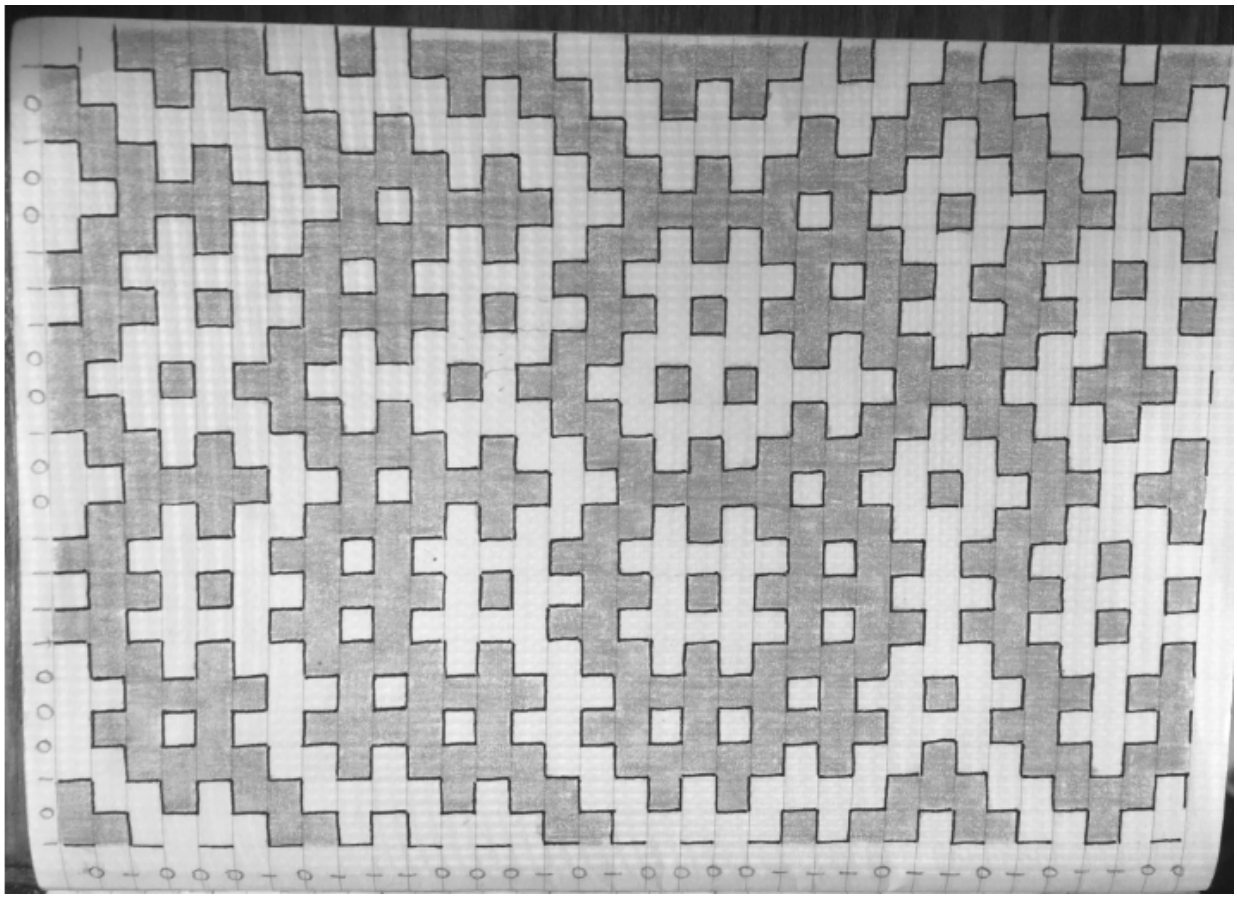
While looking at different examples, Nigel had an idea of proving that every pattern can be colored by two colors. It was based on the observation that at every grid point (not on the boundary) there is exactly one horizontal and one vertical stitch coming together. One can imagine the hitomezashi pattern to be continuing infinitely in each direction covering the whole plane and then there is no need to distinguish between points on the boundary and points inside.

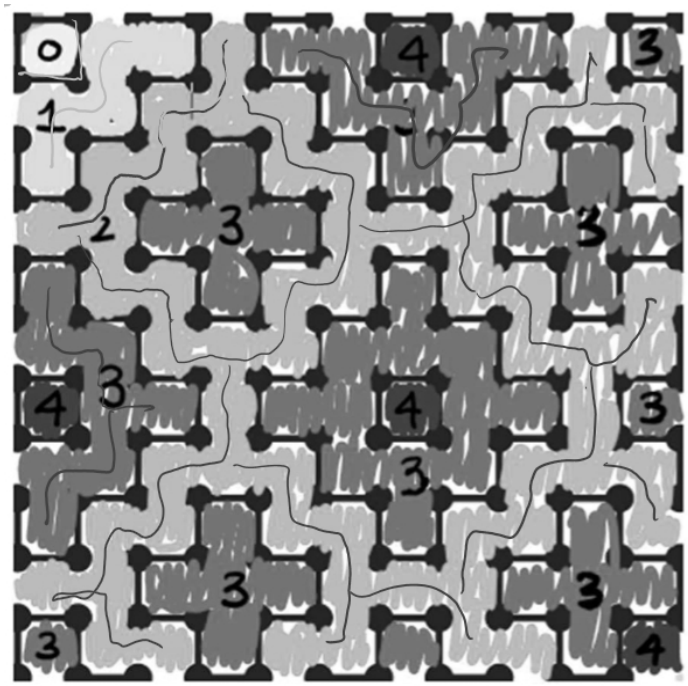
Then if we start at any point, we can follow stitches in a unique way in both directions and they form a path or strand. Sometimes a path returns to the point where it started and forms a loop. Three loops are illustrated on the left.

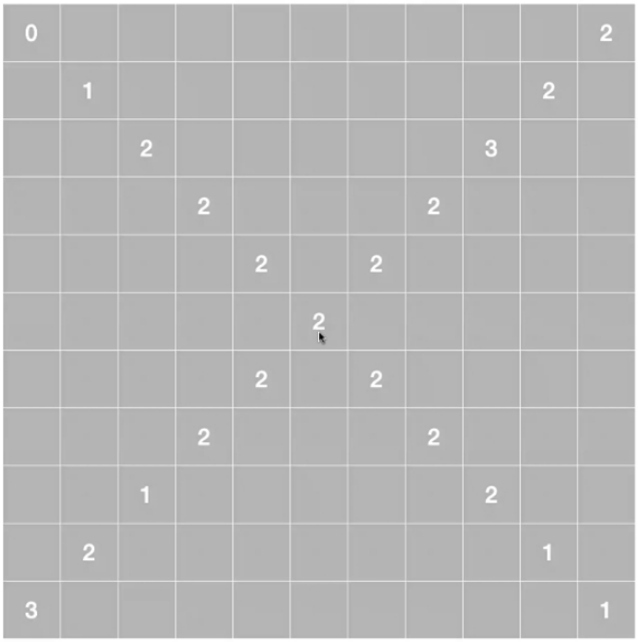
No two strands can intersect because of the property that exactly two stitches come together at each grid point. Thus the strands partition the plane into a number of regions that border each other along the strands. The pattern then can be colored by two

colors, using different colors on the two sides of a strand. An example of a colored pattern is shown below.

Nigel had another interesting insight connecting nested loops in a pattern to a topographical map. With this idea, we can lift a hitomezashi pattern into three dimensions. Start at the upper left square of a pattern, and let the region it belongs to be level 0. Then every neighboring region not yet numbered will be level 1, then every neighboring region to level 1 regions that are not yet numbered become level 2, and so on. This idea is illustrated below on the left. The lifting-up of the pattern can be done in this way because of the structure of strands we saw earlier, so this could also help visualizing that the coloring of the pattern can be done by two colors in such a way that neighboring regions have a different color.

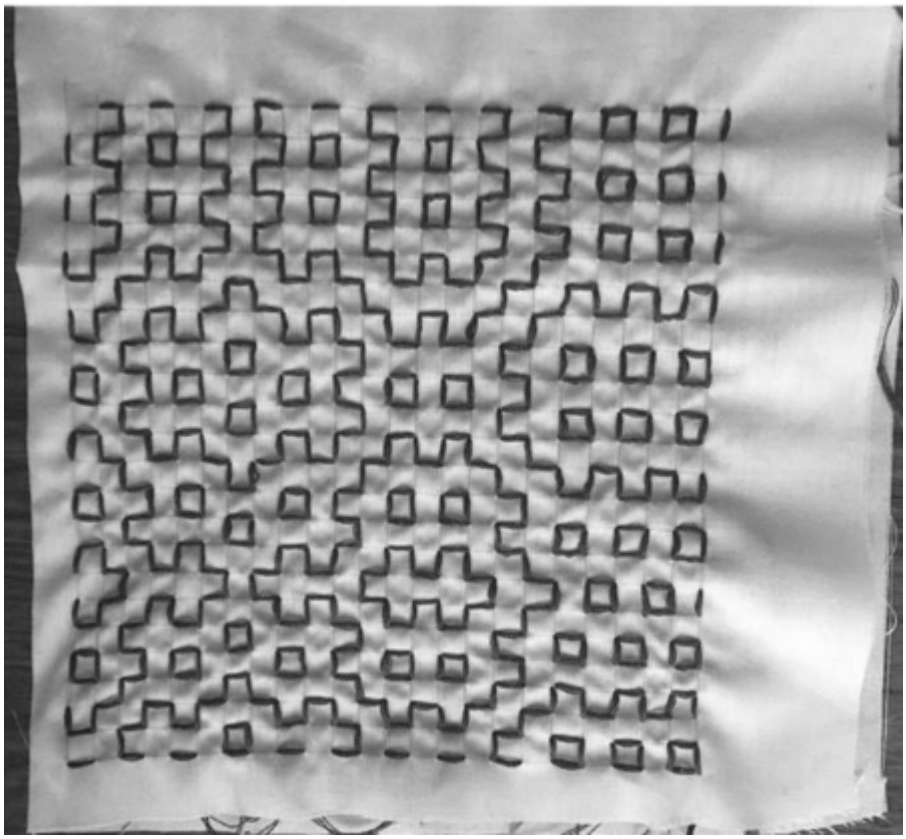




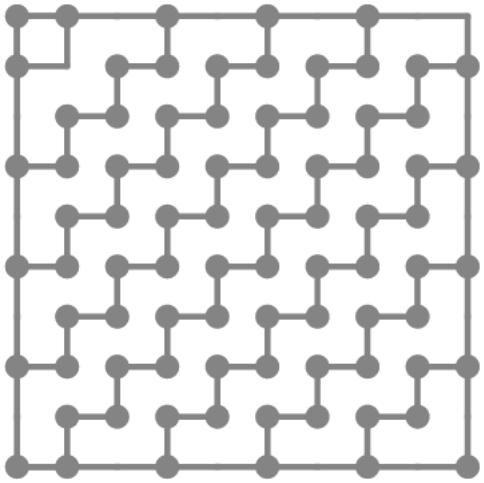


A topographical hitomezashi puzzle was invented by Gord Hamilton and we briefly talked about it. It is fun to try to find a solution, that is, a hitomezashi pattern that would allow the numbered squares to be at the indicated level. The puzzle is shown in the right above. Is there a unique solution? Gord has many wonderful puzzles on his website https://mathpickle.com/ that could bring a lot of joy and mathematical thinking to any classroom or to anyone interested in thinking about fun, open-ended problems.

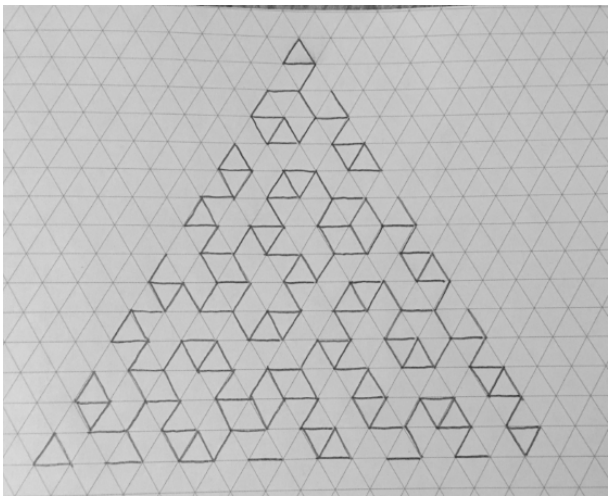
Maria brought up another interesting way of making the patterns three-dimensional by wrapping them around a cylinder or a torus. This can certainly be done if the start and the end of each grid line is different. Can a Mobius strip be made from a pattern? What would you see if you embroidered a hitomezashi pattern on fabric and looked at both sides? Below is an embroidered piece that is generated using the first 24 digits of pi.

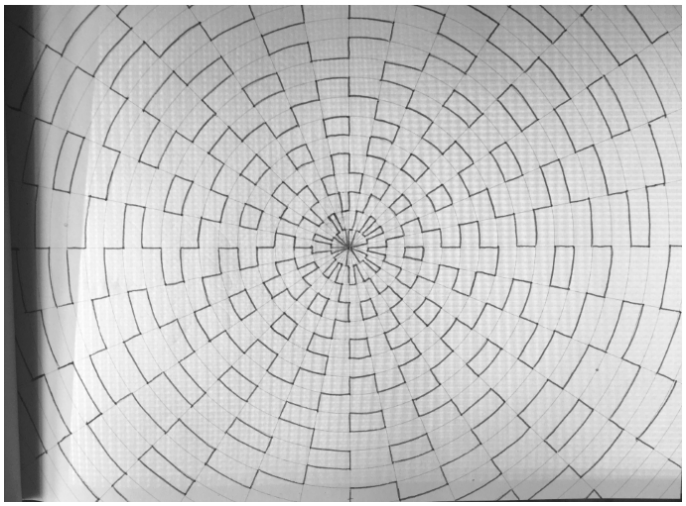


There was also a question about how many levels a certain pattern has, and whether it could be found by just knowing the strings of 0s and 1s that generate the pattern horizontally and vertically. We thought about how high the levels could go in a pattern? What is the maximum? We quickly found that in an n-by-n pattern it is possible to have n levels. The n=9 case is shown here.



It is also possible to change the grid and make hitomezashi-like patterns on different kinds of grids. A great discussion about this can be found in the Numberphile video with Ayliean MacDonald https://www.youtube.com/watch?v=JbfhzlMk2eY where a triangular grid and 0s and 1s in three directions are used. One can also consider a polar grid. (Different kinds of graph paper can be found online and printed.)





There are many interesting mathematical structures in a hitomezashi pattern. One can look for different symmetries in the design and in the individual loops and strands too. Looking at a few examples, one might notice that the loops always have a length that is even, in particular divisible by 4. More precisely, they have remainder 4 when divided by 8. Students could spend some time generating different length hitomezashi loops on a grid observing the rule that horizontal and vertical segments must alternate. We can also see that the area of the polyomino enclosed by a loop is always odd and gives remainder 1 when divided by 4. If we try to enclose the whole loop in a rectangle in the grid, the rectangle will always have odd dimensions. These results are shown by elementary methods in the paper Loops and Regions in Hitomezashi Patterns by Colin Defant and Noah Kravitz https://arxiv.org/abs/2201.03461?context=math . It would be interesting to see whether simpler proofs are possible for some of these statements. Constructing the different loops and noting the relevant constraints might help in explaining some of the results. Another good resource is the 2020 Bridges Conference paper: A Two-Dimensional Introduction to Sashiko by Carol Hayes and Katherine A. Seaton https://2020.bridgesmathart.org/workshop/ .

It is interesting to note that Japanese culture seems to prefer odd numbers versus even ones, and the hitomezashi patterns abound in them. A paper concerning even and odd numbers in different cultures can be found at https://journals.sagepub.com/doi/abs/10.1177/0270467606295408?journalCode=bsta

Share your ideas with other Bluebird Math Circle participants at **https://aimathcircles.org/Bluebird**

New Questions for Bluebird

Why is it relatively hard even for math circle people to care about proving things? Proofs? - from Maria

How big is bluebird's beak? - from Donna

BLUEBIRD SAYS—Intriguing questions! I will fly around and seek an answer. Watch this space in the next newsletters!

Submit your math-related questions at **https://aimathcircles.org/Bluebird**