BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles

Issue 29 Recap: The Game of Buzz

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Introduction

After a peaceful welcome with the song of a bluebird, we began by playing the game of Buzz.

Buzz

Buzz is a game played in a game.Members of the group count off: the first person says ‘one’, the second person says ‘two’, and so on. But if your number is a multiple of 7, you say ‘buzz’ instead.

We played the game up to 30, going around in turn. Then we looked at two diagrams of multiples of seven:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10	11
12	13	14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31	32	33
34	35	36	37	38	39	40	41	42	43	44
45	46	47	48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63	64	65	66
67	68	69	70	71	72	73	74	75	76	77
78	79	80	81	82	83	84	85	86	87	88
89	90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109	110

In the diagram at the left, numbers are listed in rows of ten. In the diagram on the right they are listed in rows of eleven. In both diagrams, multiples of seven are in red. Participants notice various patterns. MacKenzie noted that to get from one red square to another (in the diagram on the left), you move like a knight moves in chess. Nigel commented that in that diagram, you could go down two rows and right one column, or down three rows and left two columns. Donna pointed out that these moves result in diagonal lines formed by the red squares–in both diagrams. But the ‘slopes’ of the lines are different.

One exception to the “knight’s move” pattern, observable in the left diagram, If you start at the number 70 and go down two and to the right one, you fall off the right edge of the diagram. We can ‘salvage’ this by continuing from the left end of the diagram, as if the diagram ‘wrapped around’ like a cylinder. Then after 70 you arrive at the number 91, which is likewise red.

We noted that this exact pattern (two down, one to the right) does not hold when we look at the same numbers in rows of eleven. But similar patterns do hold. For example, if you start at a red square and go two down and one the right you end up on another red square. And there are other ways to describe these patterns.
Amy noted that in the left hand diagram, if you start at a red square and go ‘straight down’, you always add 70 to get to the next red square in that column. And in the right hand diagram, you always add 77. The number you add is different, but it is always the same number in either diagram.

We noted that this exact pattern (two down, one to the right) does not hold when we look at the same numbers in rows of eleven. But similar patterns do hold. For example, if you start at a red square and go two down and one the right you end up on another red square. And there are other ways to describe these patterns.

NIgel gave one explanation for the diagonal patterns: Using the table with rows of 10, moving down 2 means adding 20. Then moving across 1 means adding 1. So we've added 21 to a multiple of 7. Since 21 is itself a multiple of 7, the sum must be a red number as well. Similar explanations hold for other 'diagonal' patterns.

We briefly speculated on what would happen if we looked at multiples of 7 displayed with various row lengths (and not just 10 and 11). It seems that we will always get a 'diagonal' pattern, but that the "down" and "across" numbers will vary. Perhaps there is an interesting pattern to be observed.

We also talked about using multiples of other numbers than 7. Among other notes, we saw that if we displayed multiples of 5 on a ten-row diagram, we would not get diagonals, but vertical columns. Or can we see this as a special case of a diagonal pattern?

Buzzwhack

Next we made things harder. The game of Buzzwhack is the same as Buzz, with an added rule: if your number is a multiple of four, you say 'whack' instead of the number itself.

This is a harder game, and players had to think a bit more. We counted up to 60–which is a 'whack'. And we noted that some numbers are both 'buzz' and 'whack'. Here is a diagram showing the 'buzz' numbers in blue, the 'whack' numbers in yellow, and the numbers that are both 'buzz' and 'whack' ('buzzwhack') in green.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

We split into breakout rooms to discuss patterns in this diagram. Some patterns we noted:

Rhoda noted that the green numbers form a 'knight's move' pattern, going down 3 and left 2. In fact, the green (buzzwhack) numbers are exactly the multiples of 28.

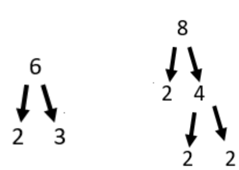
The yellow squares, the blue squares, and the green squares each form their own diagonal pattern. But we have to 'count' green as both blue and yellow, which makes artistic sense.

It also makes sense that the green numbers are multiples of 28, as 28 is the first number which is a multiple both of 7 and of 4.

We then asked what would happen if we colored all the multiples of 5 yellow (instead of multiples of 4). It was clear that then the green numbers would be exactly the multiples of 35, as this number is the first which is both a multiple of 7 and of 5.

In both cases, the first green number is the product of the two numbers generating the other colors: 28 = 4x7 and 35 = 5x7. But if we colored multiples of 6 yellow, and multiples of 8 blue, the green numbers would be multiples of 24. The product of 6 and 8 is 48, but that is not the first multiple of both.

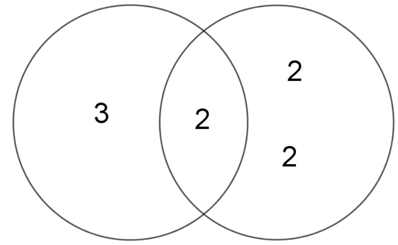
If we have a yellow number p and a blue number q, then the green numbers are multiple of the *least common multiple* of p and q. We briefly discussed how to find the least common multiple of two numbers, if we know their prime factors. For the numbers 6 and 8, we have:



So 6 = 2x3 and 8 = 2x2x2. We need three factors of 2 and one factor of 3 in the least common multiple, so it is 2x2x2x3 = 24.

The least common multiple is also the least common denominator when adding fractions.

One way to visualize this is using this diagram, which is almost like a Venn diagram for sets (except that we have many copies of the same element, because there are many copies of the prime number 2).



Finally, we looked briefly at the following table:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
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51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

This table shows buzz, whack, and buzzwhack numbers, color-coded as before. But the pairs of numbers in red are those in which 'buzz' and 'whack' numbers come one apart.

There were many interesting patterns here. There was a diagonal pattern, but it's not easy to see. We noted that sometimes the 'buzz' number comes first in the red pair, and sometimes the 'whack' number–and that these alternate.

There is much more to discover in these and other tables. Some topics to be explored are:

- Modular Systems
- Least Common Multiple
- Greatest Common Divisor
- Euclid's Algorithm
- Bezout's Identity
- Linear Diophantine Equations

These simple games lead to deep mathematical insights.

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New Questions for Bluebird

What is the best thing I can do in the summer to keep my math skills sharp?—From Donna Fernandez

A mirror reverses right for left. Why doesn't it reverse up for down as well?—From Mark Saul

Does math help in your gardening?—From Tatiana Shubin

What is your favorite way to encrypt a message? That means to translate it into a secret code.—From Nigel Wilson

BLUEBIRD SAYS—Intriguing questions. I will fly around and seek answers. Watch this space in the next newsletter!

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