BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles

Issue 18 Recap—Measuring Distances

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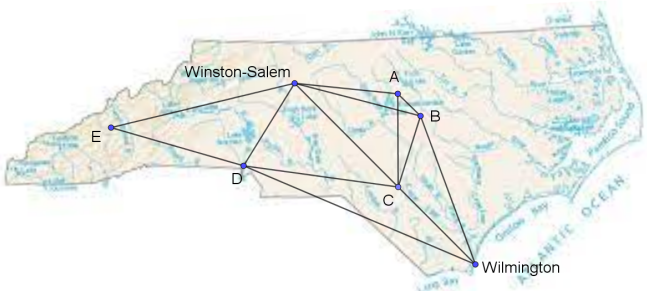
Introduction

We had a fun session exploring properties of distances on a map. Students and faculty from eight different schools were in attendance, 21 people in all.

Here is a puzzle, based on our conversations on December 13.

The map on the left shows seven cities in North Carolina. Two of them (Winston-Salem and Wilmington) are labeled. The other five are just labeled A, B, C, D, and E. One of these other five is Fayetteville. Try to identify Fayetteville, given just these clues:



i) The distance from Wilmington to Winston-Salem is 186 miles (to the nearest mile).
ii) The distance from Winston-Salem to Fayetteville is 104 miles.
iii) The distance from Wilmington to Fayetteville is 82 miles.

Measuring Distances

We started the session by looking at a mileage chart. These were once routinely included on paper maps, but have become less common with GPS maps. But we still got some important properties of distance out of our examination.

To the right is a chart of Bluebird distances between communities. Distances are given to the nearest mile. We looked at some simple questions, designed to make sure everyone knew how the information was displayed:

1) Why is the upper half blank? Is some information missing?

Lakola Cook (Magnolia Elementary School), among others, replied that the information would be duplicated if we filled in all the boxes. For example, the Bluebird Distance from Red Mesa to Ganado is given as 87 miles (entered at the intersection of the yellow horizontal and vertical lines). So the distance from Ganado to Red Mesa

(nearest mile)	Chinle	Farmington	Gallup	Ganado	Holbrook	Kayenta	Keams Canyon	Kykotsmovi	Many Farms	Newcomb	Red Mesa	Sanders	Shiprock	Teec Nos Pos	Tsaile	Tuba City	Window Rock	Winslow
Chinle																		
Farmington	84																	
Gallup	53	50																
Ganado	31	100	46								87							
Holbrook	93	167	91	66														
Kayenta	55	111	115	78	126													
Keams Canyon	42	127	83	39	62	64												
Kykotsmovi	62	146	108	61	72	62	25											
Many Farms	13	80	73	45	106	43	49	66										
Newcomb	48	40	51	59	126	90	90	110	52									
Red Mesa	57	74	105	87	148	47	91	101	41	61								
Sanders	66	121	39	36	52	115	65	86	80	81	121							
Shiprock	65	25	50	87	154	86	108	124	60	34	41	113						
Teec Nos Pos	58	51	98	86	151	65	96	111	50	49	17	118	25					
Tsaile	21	86	59	44	111	64	64	83	23	46	48	75	80	43				
Tuba City	90	169	143	97	102	68	61	36	90	137	130	122	146	129	109			
Window Rock	44	86	20	26	83	99	55	89	56	46	92	36	80	85	45	124		
Winslow	100	181	115	80	31	120	60	59	108	141	152	77	166	158	121	82	103	

should be the same (we have entered it at the intersection of the green horizontal and vertical lines).

Bluebird distance between two points does not depend on the order with which you select the points. Sometimes we write this as *distance (A,B) = distance (B,A)*. Or even just d(A,B) = d(B,A), for any two points (A,B).

2) What is the distance (given by the table) between Tsaile and Kykotsmovi?

This was not hard to read off. We select Tsaile on the left, and Kykotsmovi on the top row, and find where they intersect. The distance is 83 miles.

3) What is the distance between Gallup and Red Mesa?

This is a little trickier. If we select Gallup on the left, and Red Mesa on the top row, we are led to a blank space. So we must use the technique of problem (1), and reverse the order. The Bluebird distance will be the same. Selecting Red Mesa on the left, and Gallup up on top, we find the distance to be 83 miles.

(nearest mile)	Chinle	Farmington	Gallup	Ganado	Holbrook	Kayenta	Keams Canyon	Kykotsmovi	Many Farms	Newcomb	Red Mesa	Sanders	Shiprock	Teec Nos Pos	Tsaile	Tuba City	Window Rock	Winslow
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4) Suppose you were at Kayenta, and you wanted to read off the distances to each of the other towns on the chart. How could you do this easily? Which town is closest to Kayenta? Which is furthest?

Several students found the answer. The 'pathway' through the table is shown on the left, in orange. If we follow the path (from either end towards the other!) we can remember the lowest number we've encountered, then replace it with any lower number we encounter. That way, when we get to the end, we will have the lowest—the distance to the closest town from Kayenta. The answer is Many Farms, at a distance of 43 miles from Kayenta.

Following a similar procedure, we can find that the furthest town from Kayenta is Holbrook, at a distance of 126 miles.

5) There are some entries which should be zero, but are not shown. Where are they?

Ethan Y., in a breakout room, showed us that they were on one of the diagonals of the rectangle. The distance from a town to itself is 0 miles. Sometimes we write this as d(A,A) = 0 for any A.

These diagonal boxes are in blue in the figure above and to the left… Except for one of them, which was on the orange path. It is also a zero. But had we 'counted' it when we traversed the orange path, it would have skewed our result. In finding the nearest town to Kayenta, we would have come up with the answer 'Kayenta'. Well, technically that is correct. The Bluebird distance from any point to itself is zero. But of course the question made more sense if we interpreted it as 'Which other town is closest to Kayenta?' Not filling in these zeros lets us answer question 4 more easily.

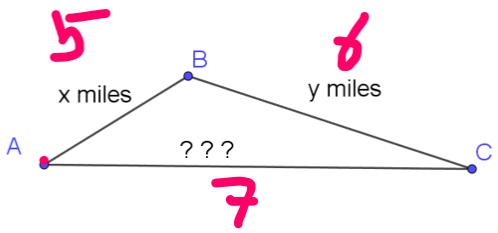
6) The distance between Newcomb and Ganado is 59 miles. The distance between Ganado and Window Rock is 26 miles. Does that mean that the distance from Newcomb to Window Rock is 59 + 26 = 85 miles? Why or why not?

Remember, these are Bluebird distances, and Bluebird can get very busy.

Several people joined in the conversation. Bernice Chatter (Winslow School) reported on work from her breakout room. She explained that it was easiest to look at the map for this one (at right).

These three towns form a triangle. A busy bluebird would not fly from Newcomb to Window Rock by visiting Ganado first—unless she had a particular reason to be in

Ganado. The point is that the distance along one side of a triangle is always less than the sum of the distances along the other two sides.



At left left) is another example that one participant drew.

If A to B is 5 miles, and B to C is 6 miles, then A to C must be shorter than 5+6 = 11 miles. Here the mapmaker chose 7 for the possible distance. But 11 is not possible, and neither is 12 or 13 or 100. They're too big to fit into a triangle.

We can write that for any three points that form a triangle, d(A,C) < d(A,B) + d(B,C).

7) Suppose the distance between point A and Point B is x miles.
Suppose the distance between point B and point C is y miles.
Is it ever true that the distance between point A and point C is x + y miles?
When might that happen?

Again, several people figured out when this happens. It happens when two conditions hold:

(1) A, B, C do not form a triangle, but lie along a (straight) line. That is, they are *collinear.*

(2) B is between A and C. Just look (picture at left, courtesy of xxxx). Here d(A,B) = 5 and d(B,C) = 6, and indeed d(A,C) = 5+6 = 11. We tried putting point B on a line with A and C, but outside segment AC. It doesn't work.

So we can revise the inequality we found in (6) to say that d(A,C) ≤ d(A,B) + d(B,C), with equality only if B is collinear with A and C, and between them.

PART II: Other ways of measuring distance.

Most of us cannot travel as the bluebird does. We cannot fly directly—unless we are piloting a helicopter. Most of the time we travel by car.

So we talked about road distances. These are distances along roads between towns. To make things a bit simpler, we considered only roads on the map we've been using.

Let's say we are staying at Kayenta—just for this discussion. Here is a chart showing distances by road and by air between Kayenta and some other towns. (All data is taken from Google Maps. Anyone can access it.)

Kayenta to	by road	by air	ratio road to air (stretch factor)
Chinle	70	55	1.3
Tsaile	95	64	1.5
Ganado	104	78	1.3
Keam's Canyon	99	64	1.5
Tuba City	75	68	1.1
Shiprock	101	86	1.2
Kykotsmovi	87	62	1.4
Window Rock	131	99	1.3

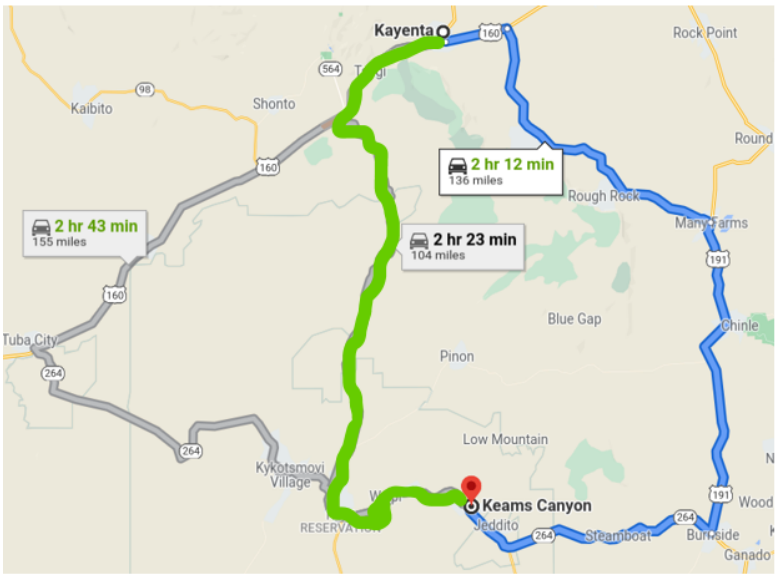
This chart is less complicated to read than the one we started with, but also has less information. Nonetheless, we were able to learn from it.

We noted that the numbers in the first column are larger than those in the second column. This makes sense, because going by air is likely to be shorter than going by road. Roads may twist and turn, may avoid hills, or cross streams at a safe place.

We talked a lot about this new 'road' distance. It is certainly still true that d(A,A) = 0: No matter how you travel, the distance from a point to itself is zero. You won't have to travel at all.

But it was a more complicated question to ask whether d(A,B) = d(B,A). It might not happen. For example, the road might be under repair in only one direction. Under 'normal' conditions, the equation would hold. In making a mathematical model of travel, we may or may not want to include these other 'abnormal' conditions.

We next talked about distance measured in minutes. Minutes? That's time, not distance. Well, but we sometimes say that "Chinle is 30 minutes north of Ganado". What we mean, of course, is that under normal conditions, it will take you about ½ hour to drive from Ganado to Chinle. We looked at how 'time distance' compared with 'road distance' using just one example:



Suppose we were planning a trip from Kayenta to Keams Canyon, and we looked at two possible routes, shown on the map at right in green and blue. The Green route looks shorter in road distance: 104 miles vs. 126 miles. But the green route is longer in time: 2 hours 23 minutes = 143 minutes, vs. 2 hours 12 minutes = 132 minutes.

We talked about why that might be the case.

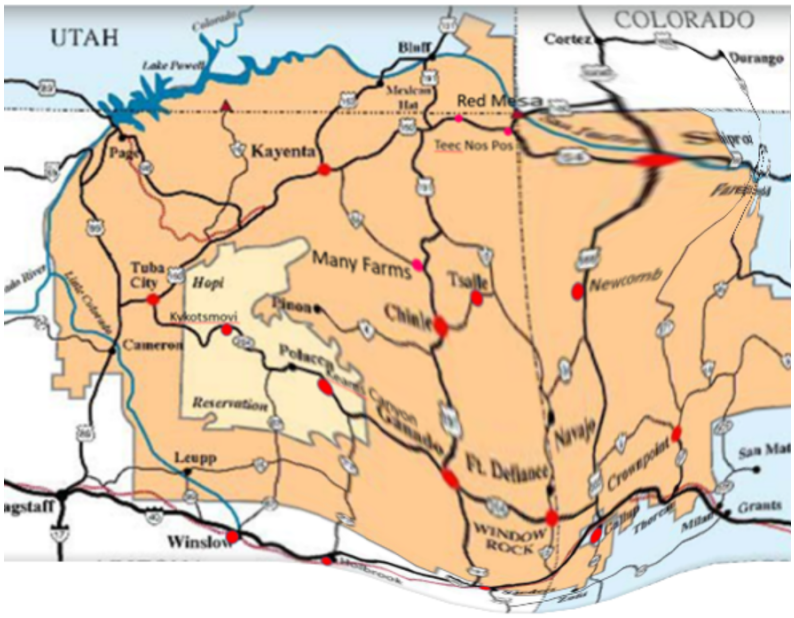
Rosebelle Nelson (Winslow School) noted that one reason might be pavement. Perhaps you can go faster along the blue route than along the green route.

Dawnlei Ben (DEAP School) gave another reason might be speed limits. The green route might go through more populated areas, where drivers must slow down. ("Don't speed! You got kids to feed!" reads one sign made by a school student in Pinon.)

Speaking of Pinon, Shaye Jones noted that there's yet another road, not shown here, from Kayenta to Chinle, over to Pinon, then down to Keams Canyon. But this beautiful road is unpaved, and takes a long time to drive. If you want to view nature, it's a nice road. But if you want to get to Keams Canyon, better choose another one.

LIkewise, it may not be true that d(A,B) = d(B,A), for all the reasons that it might not work for 'road distance'. Again, under normal conditions it will work. We may or may not want to build 'abnormal' situations into our mathematical model.

Finally, we talked about the possibility of drawing a map which represented the 'closeness' of points to Kayenta by time. It would be a warping, or distortion, of a normal map. Here is an approximation of what it might look like:



Those familiar with the area might think the map is ‘wrong’. And it may be a bit off—these are approximations. But it shows that the time to Shiprock or Window Rock is longer than expected, while the time to Tuba City is not changed all that much.

We didn’t have time to talk about the strange ways shapes get distorted when we do this sort of stretching (or shrinking). The mathematical name for it is ‘topological distortion’. And it is a deep topic indeed to study what stays the same, and what changes under such distortions.

To start off, one might think about whether we can read the lettering on the map. What stays the same, and what changes, for different letters? Does it ever change enough that we cannot read it?

Happy New Year!

Share your ideas with other Bluebird Math Circle participants at https://aimathcircles.org/Bluebird