# BLUEBIRD MATH CIRCLE

## Alliance of Indigenous Math Circles

## Issue 17

Share your problems, solutions, models, stories, and art:
**https://aimathcircles.org/Bluebird**

*I can imagine this exchange between a Western-trained mathematician (WTM) and a person raised in a traditional Native American culture (NA):*

- *WTM: What is your concept of infinity?*
- *NA: In what sense?*
- *WTM: You know, like the number of steps you need to take to travel a path with no beginning or end.*
- *NA: Oh, you clearly mean a circle.*

*Circles do play a role in indigenous notions of infinity (as something that is boundless).*

—Robert Megginson (Lacota), the Arthur F. Thurnau Professor of Mathematics at the University of Michigan in Ann Arbor.

**NEWSFLASH**

Join LIVE Bluebird Math Circle to work on these activities together with friends and family.

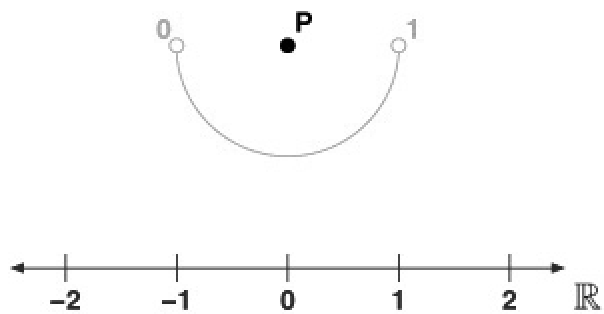
Monday, November 29, 5-6 PM MST online.

Sign up at **https://aimathcircles.org/Bluebird**

**MATH PUZZLE**

What is 3/7 chicken, 2/3 cat, and 2/4 goat?

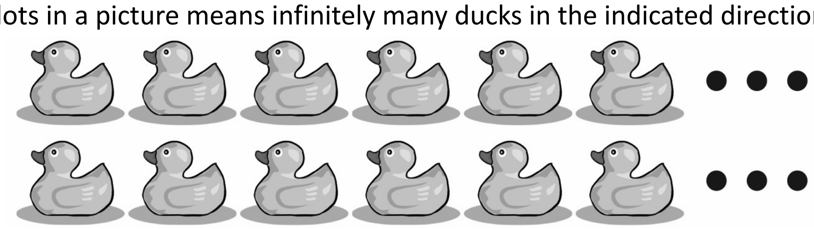
## Warm up: Are these sets of the same size?

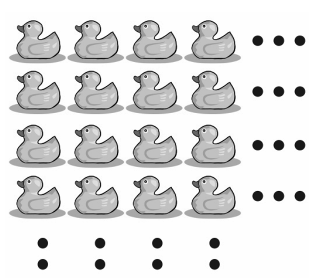




1. How come the scale in the left picture is in equilibrium?
2. Look at the right picture. Are there more points on the blue infinite line or on the green semicircle?

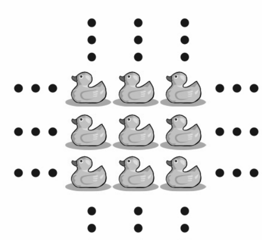
## Family Circle: Let's Count All Rational Numbers

Imagine that you have cards labeled with numbers 1, 2, 3, 4, …. — all counting numbers. Find a way to put one card on each duck via a clearly seen pattern. Do it for each of the three arrangements of ducks. Three dots in a picture means infinitely many ducks in the indicated direction.





Picture 2



Picture 3

We say that a number is *rational* if it can be written as a *ratio* of two integers, for example ½, or $\frac{-5}{8}$ , or 1 (because we can write it as $\frac{1}{1}$), or 7.333… (because we can write it as $\frac{22}{3}$). Now suppose you want to place the cards on every (positive) rational number. How would you do that?

Hint: place all positive rational numbers as the ducks in picture 2 above, or place all rational numbers as ducks in picture 3. For example, here all the positive fractions are arranged as ducks in picture 2. Can you see why some of them are in red?

1/1 1/2 1/3 1/4 1/5 …

2/1 2/2 1/3 2/4 2/5 …

3/1 3/2 3/3 3/4 3/5 …

4/1 4/2 4/3 4/4 4/5 …

⋮ ⋮ ⋮ ⋮ ⋮

Or we can place a fraction of the form a/b at the point with coordinates (a, b) in the coordinate plane, thus arranging all rational numbers as ducks in picture 3.

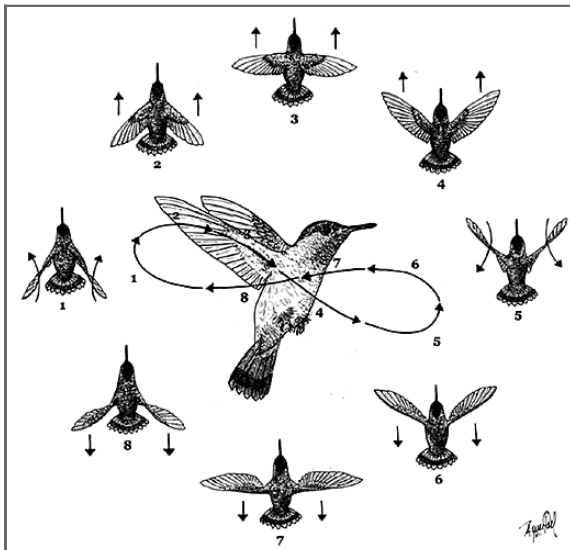
## Ask Bluebird

**QUESTION**—*How many different sizes of infinity are there?* From Craig Young , Tuba City Boarding School


**BLUEBIRD SAYS**—The answer is pretty astonishing: there are infinitely many different sizes of infinity! We have already seen that the following three sets are the same size: (a) all positive integers (whole numbers); (b) all integers, positive and negative; and (c) all rational numbers. In fact, that size is the smallest size of an infinite set. We say that these "smallest infinite" sets are countable sets. We denote this size of infinity by a Hebrew letter with zero in its subscript: $\aleph_0$ It reads, *aleph naught*.

Earlier in this flyer you've seen that a line and a semicircle have the same number of points. It turns out that there are many more points on a number line (or *real numbers*) than whole numbers — it's a larger infinity size. If you want to see why this is so, write to Bluebird at **https://aimathcircles.org/Bluebird**. We'll be happy to explain.

So we already have encountered two different sizes of infinity. What about others? Suppose we take all real numbers and form all possible subsets, such as {-3, 123/17, π}, or {0, 2021}, or {2, 4, 6, 8, …}. It turns out, there will be more of these subsets than real numbers — we get a new size of infinity. Continuing in this way, every time we find a set of a given size of infinity we can form a new set consisting of all the subsets of the previous one and this new set is guaranteed to be of a larger size of infinity. We can continue this process forever! That's how we can get infinitely many sizes of infinity.



**FUN FACT OF THE FORTNIGHT** The wings of a hovering hummingbird flutter in a specific pattern that resembles the figure-eight, the modern symbol of infinity. In many cultures, the hummingbird symbolizes eternity, continuity, or infinity.

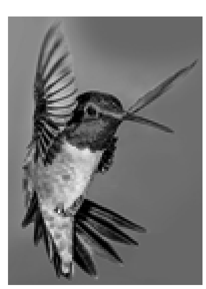


Figure-eight diagram by Aquiles Gutiérrez; hummingbird artwork by Sgaana Jaad (April White), the Haida, British Columbia islands.