BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles

Issue 16 Recap

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November 29th, 5-6 PM MST online.

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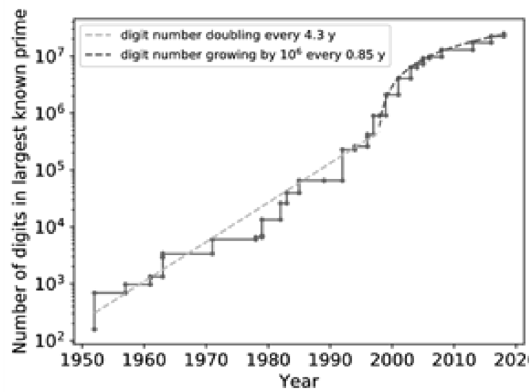
Introduction

The session started as usual with a mountain bluebird singing. But this time we had something new: Donna Fernandez provided the participants with two challenges. First, she said: “Since today’s topic is about counting and numbers, can you throw into chat the largest number that you can think of right now?”



Here are some of the answers that she got:
- “1 trillion”
- “A googol”
- “One Brazilian”
- “Googolplex”
- “The product of all numbers other people wrote in the chat”
- “Everything + 1”

What do you think about these answers? Which of them is the largest? Can you think of a bigger number? Send your thoughts to Bluebird at **https://aimathcircles.org/Bluebird**

Next, Donna told the participants to ask Bluebird any questions about numbers. And we got two excellent questions - see them at the bottom of this recap.

Strange Properties of Infinity

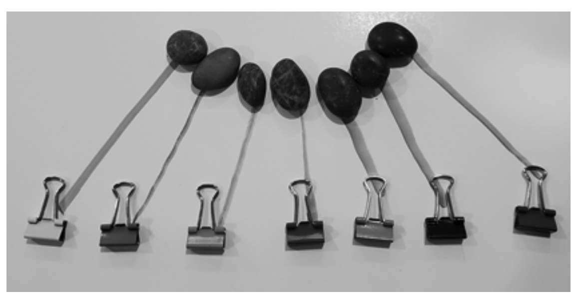
Next, we tried to help a Troll General whose species couldn’t really count beyond four but who needed to decide which of the two piles in the picture below was larger, or whether they were actually of the same size.

We briefly toyed with the idea of starting by forming groups of four in each pile, but quickly realized that there would be too many of those groups – so again, the general won't be able to count!

We struggled for some time, but then an idea came about pairing up objects from the two piles like so:

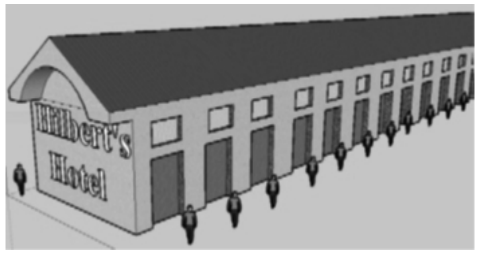
And then it becomes absolutely clear that the stone pile and the paper clip pile are exactly the same size. And we didn't need to count at all! All we did was pair up the objects in the two piles (one stone to one paper clip) and concluded that the piles had the same size. Mathematicians call this *one-to-one correspondence between two sets*. The *sets* are those piles, and we pair up their *elements*: stones in one pile and paper clips in the other pile.

What if we had two piles arranged as shown in the left picture below and for some reason we can't move them around (say, they are glued to the table)? Not too difficult – just draw lines between every stone-paper clip pair:



This last example is important – it shows that it does not matter how we actually produce a one-to-one correspondence, and also that sometimes it might take some hard thinking to achieve it.

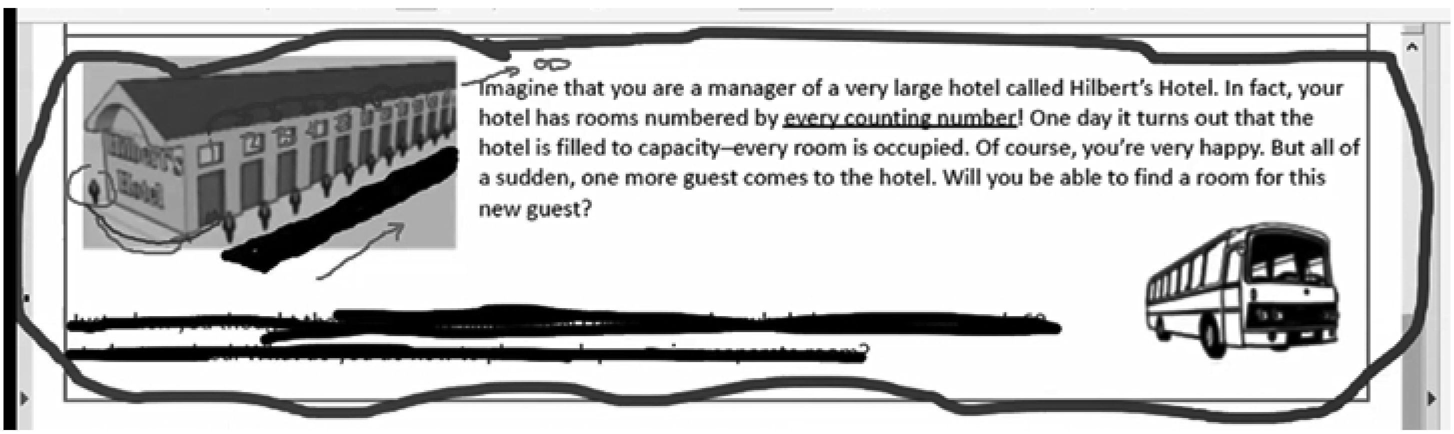
After this warm-up activity Craig Young led us to an exciting encounter with Hilbert's infinite Hotel and the many challenges faced by its manager. First, we had to imagine this hotel. It has infinitely many rooms numbered with all counting numbers – room 1, room 2, room 3, room 4, and so on never stopping – think of any counting number no matter how large, and there is a room with this number in the hotel:



Now imagine that you are the manager of this hotel, and one day you are very happy because your hotel is full to capacity – there is one person in each and every room. (See – there is a one-to-one correspondence between the rooms in the hotel and its occupants. In other words, the set of all rooms and the set of all the guests are of the same size.) And then, all of a sudden, one more person comes to the hotel and asks for a separate room. Will you be able to check this guest in?

Someone suggested doubling up some people in the hotel. But our guests are very picky and don't want to share their rooms with anybody, every person wants to have a separate room, including the newcomer.

At first the task seemed to be impossible. But then you (the manager) remembered that there is a moving walkway right in front of the hotel, starting at room 1 and extending forever:



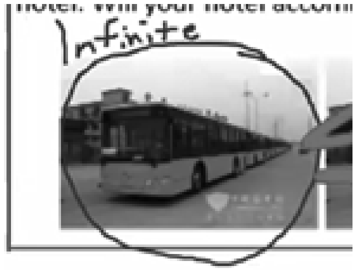
So you get a bright idea: over the PA system, you ask all the guests to take their belongings and step out of their rooms onto the moving walkway. Then the walkway moves just one room over and stops. Now the guest who occupied room 1 is in front of room 2, the guest who was in room 2 is standing in front of room 3, the guest from room 3 is in front of room 4, and so on, and so forth, forever. The guest from room 10,031 is in front of room 10,032, and the guest from room googol is in room googol+1. And now the manager asks everyone to step into the room they face – and every person has a separate room, with room 1 left empty! Thus the new guest can have that room. And again, the number of rooms is exactly the same as the number of guests despite the fact that the number of guests has now increased by one while the number of rooms remains the same. We can't help noticing this (however strange) fact since the set of all guests and the set of all rooms are in one-to-one correspondence.

So all is well in the Hilbert's Hotel – every room is occupied and every guest has his or her own room. Just at this moment, a bus brings 60 more people, and each one wants to be checked in and have a separate room. What does the incredible manager do?

At this point, the participants were quick to suggest a solution: all people step out of their rooms onto the walkway and the walkway moves 60 rooms over. Then everyone goes into the room they ended up against: person from room 1 goes into room 61, person from room 2 goes into room 62, person from room 3 goes into room 63, etc. Thus every 'old' guest is now comfortably accommodated, and the first 60 rooms are left empty. So all 60 new guests can have these rooms.

And yes, you are exactly right – the total number of rooms has not changed, the number of people has increased by 60, and yet the sizes of these two sets are still equal to one another. Strange properties of infinity, indeed!

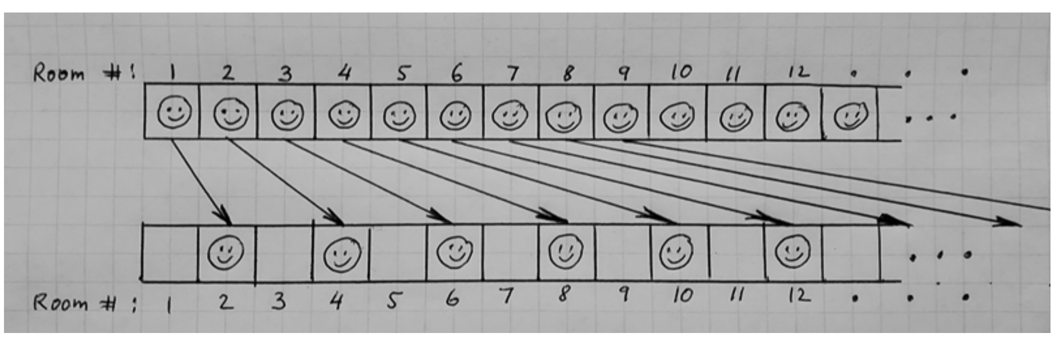
While the manager was busy with those additional 60 people, something else happened.



A bus with infinitely many people came to the hotel: the bus has seats numbered 1, 2, 3, 4, 5, …. – there is a seat with every counting number – and each seat is occupied with a passenger. All these people want to check into the Hilbert's Hotel, and each one wants a separate room! This seems to be a difficult situation, and Craig decided that it was time to go to breakout rooms for separate discussions.

When we came back together again we exchanged several ideas. One idea was to build one more infinite hotel. And of course it would work, but the problem is that all these bus passengers are tired and they want a hotel room tonight. So maybe something else could be done? Well, the answer is yes. Beth Cammarata suggested a nice solution.

She said, "Let person in room 1 go to room 2; person in room 2 go to room 4; person in room 3 go to room 6; in general, let person in room n go to room 2n, so everyone doubles their room number." See the diagram below:



Now every even-numbered room is occupied by an 'old' guest, while every odd-numbered room is empty, and there are enough empty rooms to accommodate each and every passenger in the infinite bus.

There was one more problem in Issue #16: What if the hotel is full to capacity and infinitely many buses, each bus with infinitely many passengers, arrive? Would it be possible to place all these new people in the fully occupied hotel so that every person gets a separate room, while all people already in the hotel remain there and also have their separate rooms?

At the meeting, we ran out of time. Could you help the manager to solve this last problem? Let us know what you think. And come to the next Bluebird MC meeting on Monday, November 29, where we'll discuss this problem and others.

Share your ideas with other Bluebird Math Circle participants at **https://aimathcircles.org/Bluebird**

New Questions for Bluebird

Where do numbers come from? – from Mae A.

In other languages, like Navajo, what's the largest number that's not borrowed from another language? – from Mark Saul

BLUEBIRD SAYS—Curious questions. I will fly around and seek an answer. Watch this space in the next flyers!

Submit your math-related questions at **https://aimathcircles.org/Bluebird**