



# BLUEBIRD MATH CIRCLE

## Alliance of Indigenous Math Circles

### Issue 16

Share your problems, solutions, models, stories, and art:  
<https://aimathcircles.org/Bluebird>

*I would encourage students to actively participate in STEM events... Keep on being curious. Don't be afraid to ask questions. Keep learning. Set goals for yourself. You can do it! You can work for NASA too!*

—Raquel Redhouse, member of the Navajo Nation, a NASA Engineer

Join LIVE Bluebird Math Circle to work on these activities together with friends and family.

#### NEWSFLASH

Monday November 15, 5-6 PM MST online.

Sign up at <https://aimathcircles.org/Bluebird>

Place some +’s between eights to make a true equation:

#### MATH PUZZLE

8888888=1000

## Warm up: Help a Troll General to Count



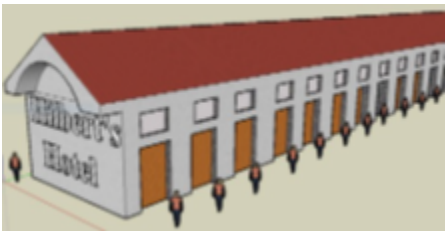
*Discworld* created by an English writer Terry Pratchett is inhabited by many different sentient species, such as Trolls, Dwarfs, Elves, and Gargoyles. All these various peoples have their own languages and cultures. Trolls on the Discworld are, essentially, living, mobile rocks. Consisting largely of silicon, Discworld Trolls vary in intelligence depending on their body temperature (as silicon heats, it loses efficiency; thus, 'keeping a cool head' is a literal fact of Troll existence). Trolls have a numeral system of their own, with the base numerals being one (1), two (2), many (4), and lots (16). How they continue from there is unknown, even to some of them. For

example, Jade in *“Monstrous Regiment”* says she *“ran outa numbers”* at that point. Trolls and Dwarfs have fought many battles, with huge numbers fighting on both sides. Imagining that a Troll general wants to figure out whether he has more soldiers than Dwarfs do. How could he do it?

## Family Circle: Strange Properties of Infinity

Let’s ask questions about whole numbers, also called *counting numbers*: {1,2,3,4,5,6,...}.

- How many counting numbers are there?
- Can we name every counting number?
- Can we write down every counting number?
- Can we find the largest counting number?



Imagine that you are a manager of a very large hotel called Hilbert’s Hotel. In fact, your hotel has rooms numbered by every counting number! One day it turns out that the hotel is filled to capacity—every room is occupied. Of course, you’re very happy. But all of a sudden, one more guest comes to the hotel. Will you be able to find a room for this new guest?



Just when you thought that everybody had been accommodated, a whole bus with Mr. Young’s 60 students arrived. What do you do now to place each person in a separate room?

It actually turns out that Mr. Young now has infinitely many students! What do you do now?

And finally, when you thought that all is well, infinitely many more busses, each having infinitely many students, arrive to your hotel. Will your hotel accommodate all the new arrivals, placing each person in a separate room?



## Ask Bluebird

**QUESTION**— *Can a triangular number be a square number?* - from Mark Saul

[See Flyer #14 and Community Recap #14 to learn about triangular, square, and other figurate numbers.]

**BLUEBIRD SAYS**—The answer is yes!

To begin with, 1 is the first triangular and also the first square number.

The next smallest such number is 36, which is the 8th triangular number (check it!) and the 6th square number, and the next one is 1225, the 49th triangular and 35th square number.

In fact, there are infinitely many such numbers. Let's see how we can try to find them all.

The  $n$ th triangular number is equal to  $\frac{n(n+1)}{2}$ , and the  $m$ th square number is  $m^2$ . So we need to find all pairs  $(n, m)$  of whole numbers  $n$  and  $m$  that satisfy an equation  $\frac{n(n+1)}{2} = m^2$ . This is an example of a so-called *Pell's Equations* studied in a branch of mathematics named Number Theory.

Triangular square numbers are helpful for creating knit blankets like this one:



© Pat Ashforth & Steve Plummer of Woolly Thoughts. See <https://www.ravelry.com/patterns/library/data-log>



### FUN FACT OF THE FORTNIGHT

The number *googol* is  $10^{100}$  (100 copies of 10 multiplied together or 1 followed by 100 zeros).

*Googolplex* is  $10^{\text{googol}}$  (googol copies of 10 multiplied together or 1 followed by googol 0's). There isn't enough room in the universe to write down the number googolplex, even if each 0 was the size of an atom.