BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles

Issue 9 Recap

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NEWSFLASH Join LIVE Bluebird Math Circle with friends and family.

August 9th, 5-6 PM MDT online.

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Introduction

After a peaceful welcome with the song of a bluebird, we began a discussion with the group. Marie Brodsky, who is currently a math student at the University of Maryland, was leading the session for the first time, so she asked each participant to introduce themselves and one of their hobbies.

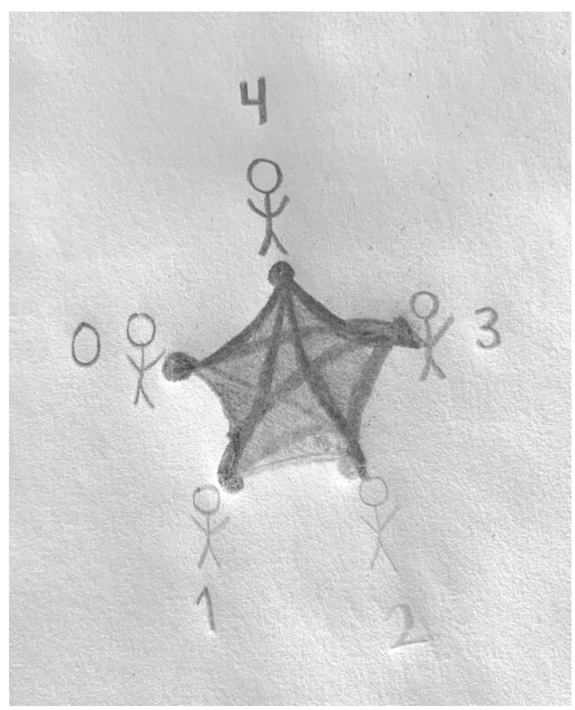
It was fun to learn more about each person who came to the math circle! We learned that several students were artists, and kept this in mind throughout the math circle as we discussed the creative approaches that could be taken to solve different problems! Several students kindly duplicated their spoken replies in the chat in case any others were having audio difficulties—everyone appreciated this thoughtfulness.

Handshakes and Combinatorics

We dived in right away with a puzzle! Before drawing anything to share on the screen, Marie asked everyone to think about what their instincts are when they hear the problem:

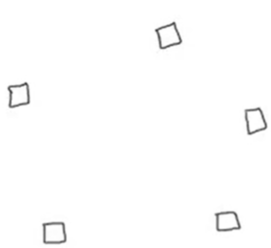
> *There are five friends, and they have not seen each other for a very long time in person, because it has been everything over zoom, which I think you all can relate to. And finally, eventually, the pandemic ends, and everyone gets back together. And since they haven't seen each other in so long, they want to handshake with each other or elbow bump or whatever it is. And you're going to figure out how many handshakes are going to take place, just between those five people. So my question to you is, how would you start approaching that problem?*

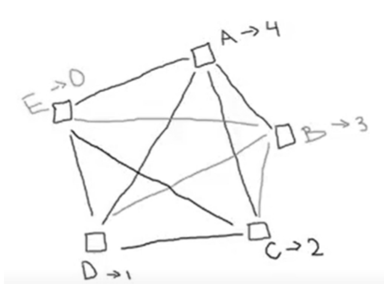
Some suggestions were also given: "I know that several of your artists, so maybe you think about how you could draw it and do it that way? Or maybe you think about acting it out?" Marie proposed that we begin by drawing a diagram to look at. "So I'm just going to draw little squares for people… You're going to have to use your imagination to pretend they're people."



Lila A. later emailed Bluebird the artistic diagram on the right.

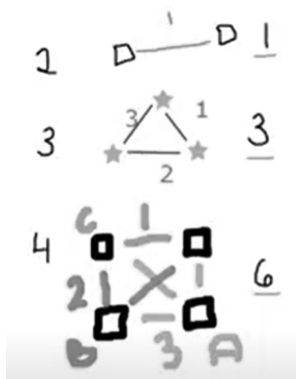
After drawing the people, we named them A, B, C, D, and E so that we could easily refer to each person individually. We also needed a way to represent the handshakes that take place between them, so we used line segments connecting them.



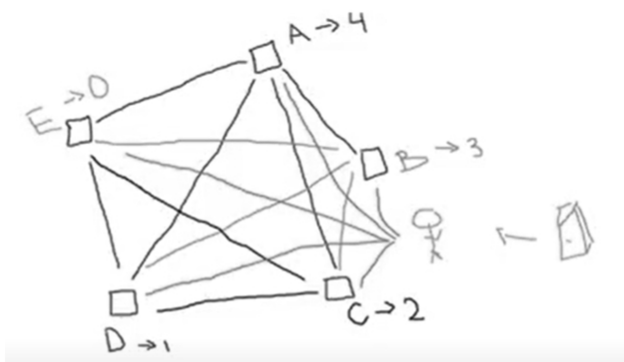


We thought about it like this: if person A goes first, they'll shake hands with everyone but themselves, meaning they'll do 4 handshakes. Then, person B will shake hands with everyone except themselves and A (because A already shook hands with them!) so they will have 3 handshakes to do. Continuing this way, C will have 2 more handshakes, D will have 1, and E will not have to initiate any handshakes because everyone else will have already come to shake hands with them. So, in total, we calculated that 4 + 3 + 2 + 1 + 0 = 10 handshakes will take place among the 5 friends. We summarized this method as one where we "think from each person's perspective" about how many handshakes they'll need to start.

So now that we figured it out for 5 friends, the question was: could we figure out a way to calculate the number of handshakes among 10 or 100 friends? To figure out if we might be able to find a pattern, we started with even smaller groups. After sketching out two people having just one handshake to do, Marie asked one student to draw the diagram of handshakes for a group of 3, and another to draw the diagram for a group of 4.

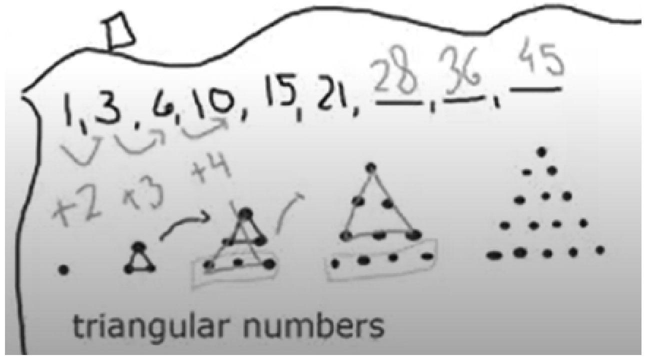


Here, we saw that 3 people would have 3 handshakes, and 4 people would have 6 handshakes. To gain insight into the pattern, we imagined: "What if we take the five people and we just have their sixth friend who was always late to everything before the pandemic and is still late to everything, run into the room. And they're like, "Oh no," because they haven't shaken hands with anyone yet. How many people do they have to shake hands with? "



In our updated diagram with the new friend running in, we saw that the friend has to do 5 handshakes: one with each person already in the room. So our calculation went from 4 + 3 + 2 + 1 + 0 total handshakes to 5 + 4 + 3 + 2 + 1 + 0 total handshakes.

Looking at the sequence we got: 1, 3, 6, 10, 15… handshakes as the number of people in the room increased. We noticed that we can get from 1 to 3 by adding 2, from 3 to 6 by adding 3, from 6 to 10 by adding 4, from 10 to 15 by adding 5, and so on. This way, we figured out that the next number would be 15 + 6 = 21, and the one after would be 21 + 7 = 28.



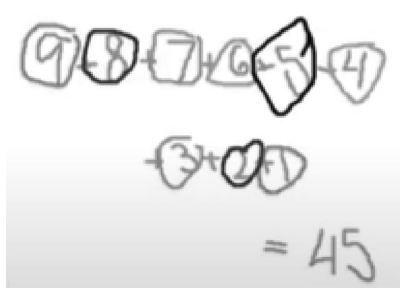
Once we figured out the pattern in the numbers, we also saw a cool way that this pattern can be seen geometrically. Representing the numbers in the sequence as little dots, we saw that they form triangle shapes. Marie explained that these are known as the "triangular numbers" because of this neat property that they have. Each newly added bottom row of the triangle can be thought of as the number of handshakes the new friend running in has to do!
We then tried a totally new approach to this problem, that at first made it seem like we broke math!

> *Okay, there are five people. How many handshakes will each person do? Well, Person D isn't doing any fewer handshakes than Person A, they just start doing them in a different order. Right? So each person is going to do four handshakes. So we have four handshakes, times the five people in the room. Which is 20. This seems like a much simpler way of doing it right? But the problem is it doesn't match what we got before. Because before we got that there were 10 handshakes taking place. So what's going on?*

Since it seemed like math was broken, we decided to just write out all the possible handshakes and see which answer was really correct. In our list, "BD" for example means the handshake between Person B and Person D. We counted 20 entries in our final list. But one student pointed out a problem: some of the handshakes were duplicates! For example, if B already shook hands with D, then D wouldn't need to shake hands with B again. To account for this, we went through the list and crossed out all the duplicate handshakes. We were left with half of 20, which was 10 as we had calculated with the previous method. Phew, everything worked out!

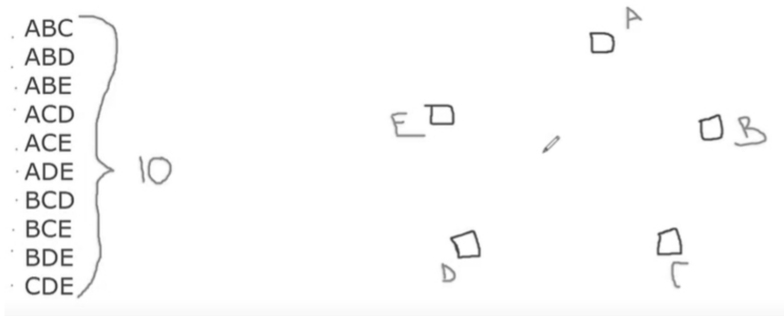
AB	~~DA~~
AC	~~DB~~
AD	~~DC~~
AE	DE
~~BA~~	~~EA~~
BC	~~EB~~
BD	~~EC~~
BE	~~ED~~
~~CA~~	
~~CB~~	
CD	
CE	

Now that we felt confident with both approaches, we decided to try increasing to 10 people and trying each method. With the first method, we got 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1, which was easier to calculate after we discussed a trick for "pairing up" the numbers in the list that added up to 10 (color coded) to make the addition easier.

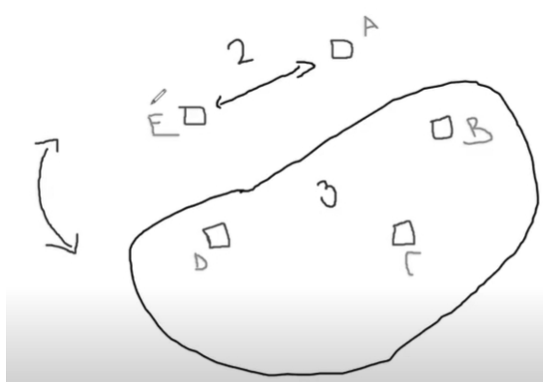


With the second method, we got 10 people × 9 handshakes per person = 90, but remembered that half of these would be duplicates that we shouldn't count, so we ended up with 90 / 2 = 45 handshakes, same as given by the first method :)

As a last challenge, we returned to our original 5 friends and thought about what would happen if they wanted to also do group hugs with 3 people each. How many unique group hugs could they make with every combination of 3 people? Since we weren't sure how to start on this one, we wrote out the unique possibilities. But it was weird—we got 10 combinations, just like with the handshakes! Was this a coincidence or was there a reason this was happening?



After exploring this question a bit more, we figured out a way to think about it that helped understand why the number of combinations ended up the same. We imagined that whenever 3 people did a group hug, the 2 remaining friends took the time to shake hands. Or vice versa, whenever two people decided to handshake, the remaining 3 took the time to hug. This way it becomes clear that the number of possible groups of hugs has to be equal to the number of handshakes!



To solidify this idea using another example, Marie suggested that we think about cookies. Suppose we baked 10 cookies and are trying to pick 2 to take with us. How many ways are there to do this? What about if we want to pick 8 cookies to take? "Think about like we thought of the hugs and the handshakes. When we're picking eight cookies to take with us, all we're doing is we're saying okay, you two remaining ones are staying here. Or when we're picking two cookies to take, we're really telling the other eight cookies, you're not going anywhere—you're staying on the table." So the number of ways to pick 2 cookies or 8 is the same, because 2 + 8 = 10 just like with the friends we had 2 (shaking hands) + 3 (group hugging) = 5.

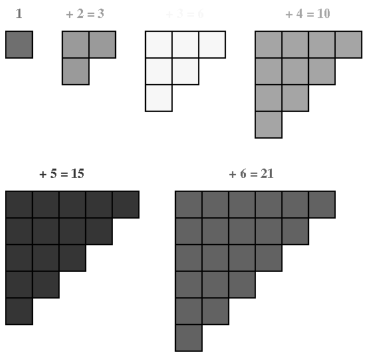
To wrap up, we went around and heard from each participant. "Do you think anything was particularly cool? Or confusing?"

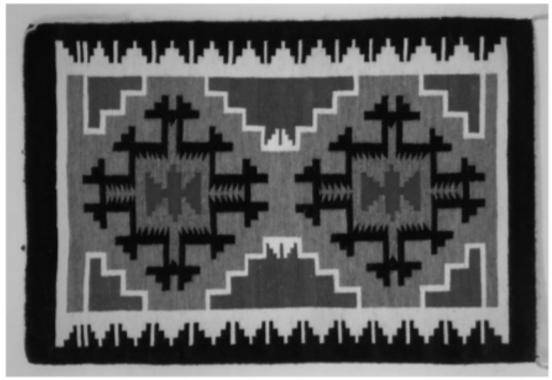
One student shared that "I think it was interesting that even if it was a group of three, like doing a group hug had the same number [as handshakes] was also a bit confusing, too." Sometimes cool topics are surprising so they aren't easy for our minds to figure out right away! We discussed that this was totally okay to feel.

Another participant said "I find it interesting how I can show how everything isn't always how it seems [...] and also how different perspectives can change things entirely. That's like really tripping me up." Haha, yes, we definitely saw several angles, even when we first thought about it from every individual perspective, versus thinking about it as zoomed out."

Someone also commented on the pattern we saw: "I like how visual those numbers are when you draw those triangles. I wonder who came up with such a nice name, triangular numbers?"

This question inspired us to look at some Native American designs with staircases that followed the pattern of the triangular numbers. Beautiful designs—we also explored them in the “Chessboards and Tiling” session (see Community Recap #4).





Images: Elena Stanescu Bellu, Navajo rug, Hopi pot by Ethel Youvella

We wrapped up by discussing these and then talking with one of the students about how cool it would be to artistically model some of what we did today in a 3D software.

Share your ideas with other Bluebird Math Circle participants at **https://aimathcircles.org/Bluebird**

New Questions for Bluebird

What do the sound waves of the bluebird look like? – from Marie Brodsky

BLUEBIRD SAYS—Curious question. I will fly around and seek an answer. Watch this space in the next flyer!

Submit your math-related questions at **https://aimathcircles.org/Bluebird**