BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles

Issue 6 Recap

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Monday June 21, 5-6 PM MDT online.

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Introduction

In its usual way, Bluebird's song called us to gather and led to yet another question, posted at the end of this recap.

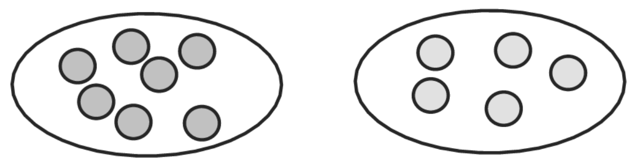
We began by looking at the Issue 6 flyer and the quote from Donna Fernandez, "Math is Indigenous," as a reminder that mathematics belongs to everyone and that everyone can experience the pleasure of doing mathematics. We also noted that there is a new website, IndigenousMathematicians.com, that profiles Indigenous mathematicians and provides resources for learning more about them. We want everyone to know that mathematics is a natural part of being an Indigenous person and that there are other Indigenous people who love math and use it in their lives every day.

We then turned our attention to the recent flyer, which described a game that we started to play.

Family Game Time

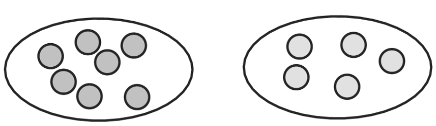
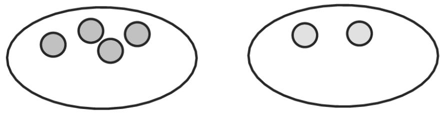
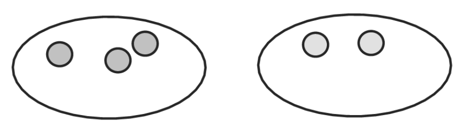
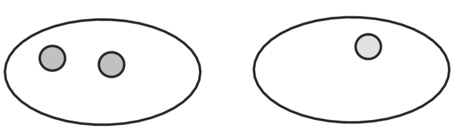
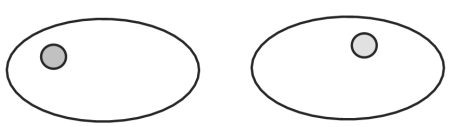
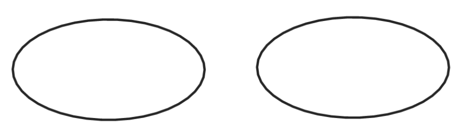
To play the game, we start with two piles of stones, which we represented on a shared Jamboard, and allow players to take alternating turns. For their turn, a player could

- Take as many stones as they wanted from a single pile, or
- Take the same number of stones from both piles.



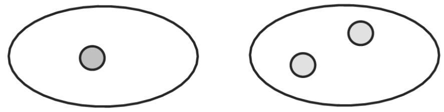
The winner is the player who takes the last stone. You can make your own copy of the Jamboard from the one at gvsu.edu/s/1JQ.

We started by playing a game together that went like this:

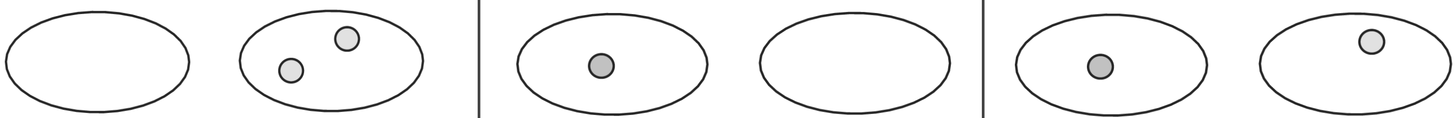
Start	First move	Second move
		
Third move	**Fourth move**	**Final move**
		

After this, we went into breakout rooms so that we could play the game some more and make note of any things that we noticed. Before each game, we can choose how many stones there are in the two piles.

The Jamboard encouraged us to play a game where there were two stones in one pile and one in the other.

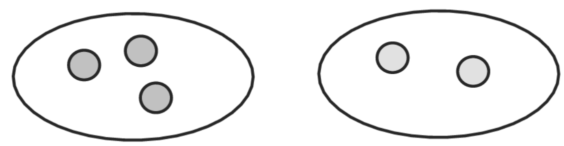


The person who plays first only has three possible moves:



One student noticed that the second person will be able to win no matter which move the first person makes. So we noted that the situation with two stones in one pile and one stone in the other is a losing position for the person who makes the next move. As shorthand for this position, we wrote (2, 1).

By playing some more games, a few people noticed that we would win if we could leave the other player with the (2, 1) position. For instance, if we started with (3, 2), then we could take one stone from each pile, which leaves the other player with (2, 1). We were then guaranteed to win.



As we looked at games that began with more stones, one student said they found it challenging because they were having a hard time thinking about it “in the forward direction.” This gave us the idea to try to think about the game in the “backward direction.” So, for instance, if we want to leave the other player with (2, 1), what are some possible positions with which we could begin our turn? For instance, we could start with (3, 2) because we had seen that we could remove one stone from each pile. In the same way, we saw that we could start with (4, 3) because we could remove two stones from each pile to leave the other player with (2, 1). So (3, 2) and (4, 3) are guaranteed to be winning positions for the next player.

Someone noticed that if we started with (3, 1), then we could take one of the three stones to reduce to the (2, 1) situation so that (3, 1) was a winning position. In fact, as long as one of the piles had only one stone, such as (4, 1), (5, 1), and so on, we could leave the other player in the (2, 1) position and be guaranteed to win the game.

In this way, we saw that there are lots of winning positions. We know that (2, 1) is one losing position for the next player. A good way to think about the game is to continue looking for losing positions and see if there is a pattern in them. For instance, is (5, 3) another losing position? If you think about this game long enough, you can discover a relationship to the golden ratio, which is about 1.618…, a special number that appears throughout mathematics.

It was fun to see how the student’s comment about thinking in the “forward direction” led us to think about the game differently. Rather than imagining our next move and how the other player might respond, we started to strategically work backwards from the losing position that we had already identified.

One of the students said, “That was interesting and interactive!”

A New Question for Bluebird

What are polynomials and monomials?—from Ye-Shiao T.

BLUEBIRD SAYS—Let me search for an answer and report back in a future flyer!

Submit your math-related questions at **https://aimathcircles.org/Bluebird**