BLUEBIRD MATH CIRCLE
Alliance of Indigenous Math Circles
Issue 14

Share your problems, solutions, models, stories, and art:
https://aimathcircles.org/Bluebird

All dreams spin out from the same web.

- Hopi Tribe

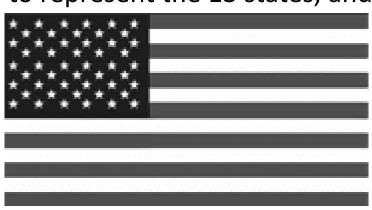
NEWSFLASH Join LIVE Bluebird Math Circle to work on these activities together with friends and family.
Monday October 18, 5-6 PM MDT online.
Sign up at **https://aimathcircles.org/Bluebird**

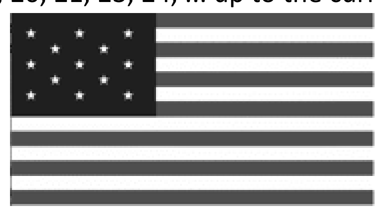
MATH PUZZLE Why most buildings in the world don't have the 13th floor?

Warm up: Figurate Numbers

From the time immemorial mathematicians have always been interested in the properties of numbers that can be arranged into regular shapes such as a triangle or a square.

An excellent example of the problems of arranging a given number of dots into a pleasing shape is the American Flag. The decision was made early on that each star in the "stars and stripes" should represent one of the States in the United States. This has changed often over time and the flag has been redesigned on many occasions. The first with this format was designed in 1777 and had just 13 stars to represent the 13 states, and then was extended to 15, 20, 21, 23, 24, ... up to the current 50 stars.



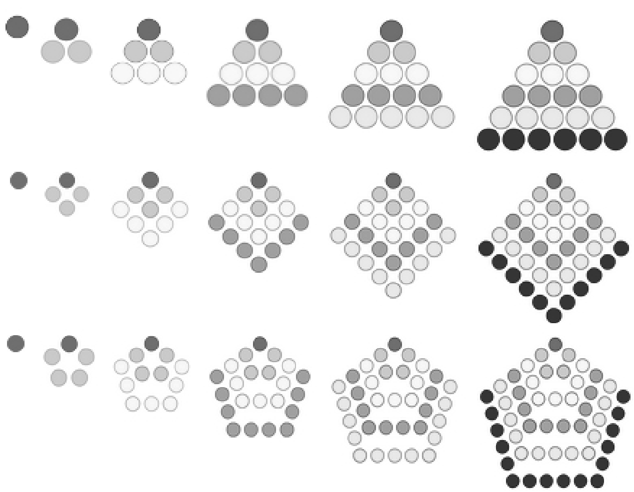


Questions:

1. How many stars does the left flag contain? When was this design adopted?
2. Same questions about the right flag.
3. Suppose one new state was to be added to the United States, making 51. How would you redesign the stars to make a nice pattern?
4. Same question about two new states: how would you redesign 52 stars to make a nice pattern?

Family Circle: Polygonal Numbers

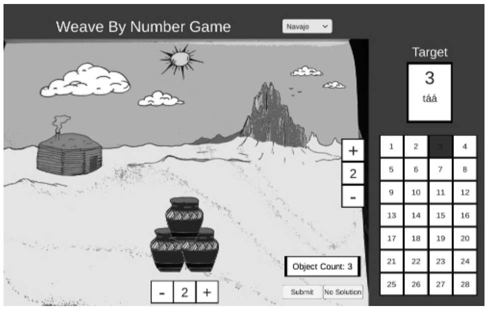
Numbers that can be represented by dots arranged in shapes of regular polygons are called *polygonal* numbers. Here are examples of the first few *triangular*, *square*, and *pentagonal* numbers:

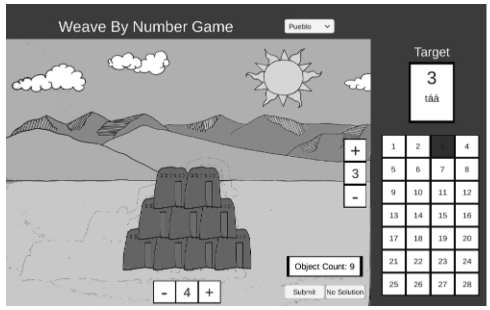


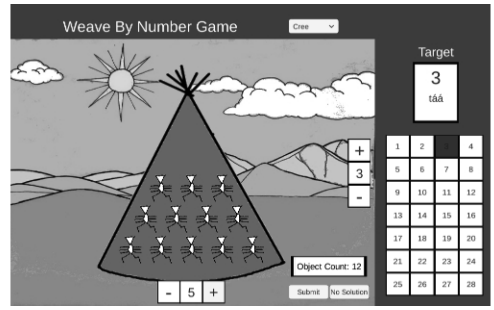
Questions:

1. The first six triangular numbers are 1, 3, 6, 10, 15, 21 (see the picture on the left). What is the 7^{th} triangular number? The 8^{th}? The 100^{th}?
2. What are the first six square numbers? What are the 7^{th}, 8^{th}, 100^{th} square numbers?
3. What are the first six pentagonal numbers? What are the 7^{th}, 8^{th}, 100^{th} pentagonal numbers?
4. Every square number is the sum of two triangular numbers. Can you see why?
5. The polygonal numbers are constructed with 'flat dots' arranged into flat polygons. Can you construct 3D analogs for numbers constructed of little spheres arranged into 3D shapes such as pyramids or cubes?

Flat figurate numbers can be of the shapes different from regular polygons. For example, *rectangular* numbers or *trapezoidal* numbers. Donna Fernandez and students from Navajo Preparatory School created a computer game about trapezoidal numbers - **https://roshan2205.github.io/Weave-By-Number/**







The game is called *Weave by Number*. The goal of the game is to create a geometric weaving pattern found on Native American rugs. The object of the game is to stack objects to match the target number to create a rug design. To play, select a number on the rug, then use the + or - controls to change the height and width of the object. The challenge is that there may be no solution, one solution, or many solutions. If the number of objects in the stack matches the target number, then click "Submit". If no solution can be found, then click “No solution”. If correct, the number on the grid will change color.

When you click on the numbers, they will be pronounced in various indigenous languages - Navajo, Pueblo, and Cree. This is a great way to learn an indigenous language. The game is designed with artwork depicting the landscape of the tribal lands and symbols or objects from the respective tribes.

Ask Bluebird

QUESTION— *How come there are algebra 1 and algebra 2?* - from Ye-Shiao T.

BLUEBIRD SAYS— The short answer: because there is too much algebra to learn for most students to do it in one year.

A deeper answer: We can think of (school) algebra in several ways. One way is as a generalization of arithmetic. We can notice a pattern in arithmetic such as:

$3 \times 5 = 14 = 4 \times 4 - 1$
$4 \times 6 = 24 = 5 \times 5 - 1$
$5 \times 7 = 35 = 6 \times 6 - 1$
$6 \times 8 = 48 = 7 \times 7 - 1.$

But the single algebraic statement $(x - 1)(x + 1) = x^2 - 1$ expresses all of them at once. That conception of algebra applies to most of Algebra I.

In Algebra 2, we expand this idea of what algebra is. We study algebraic expressions for what they are, often without thinking what they tell us about numbers. So, for example, we know that:

$x^2 - 1 = (x - 1)(x + 1)$
$x^4 - 1 = (x^2 - 1)(x^2 + 1)$
$x^2 - 100 = (x - 10)(x + 10)$
$(x + 3)^2 - (x + 2)^2 = [(x + 3) - (x + 2)] \times [(x + 3) + (x + 2)] = 1 \times (2x + 5).\ (Try\ it!)$

We can express all this by saying $A^2 - B^2 = (A - B)(A + B)$, not just when A and B are numbers, but when A and B are themselves algebraic expressions.

FUN FACT OF THE FORTNIGHT	Cicadas incubate underground for long periods of time before coming out to mate. Sometimes they spend 13 years underground, sometimes 17. Why? Both those intervals are prime numbers and biologists now believe cicadas adopted those life-cycles to minimize their contact with predators with more round numbered life-cycles.