Join LIVE Bluebird Math Circle to work on these activities together with friends and family.

**NEWSFLASH**

Monday September 27, 5-6 PM MDT online.

Sign up at https://aimathcircles.org/Bluebird

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**MATH JOKE**

by Izzy Ehnes

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**Warm up: Ashbii Navajo Stick Game**

The traditional Navajo game of Ashbii is played with three sticks.

The *tsi*i stick is all black on one side, and half black on the other side.
The *nezhi* stick is half red on one side, and half black on the other side.
The *tqelli* stick is all red on one side, and all black on the other side.

The sticks are tossed upwards, so they bounce off a buffalo hide (or just a blanket) stretched above the players. Then they fall into a basket so that they lie flat. (A stick that doesn’t land in the basket doesn’t count in the game at all.) Players get different numbers of points depending on how the sticks land.

Here is one example: The three sticks have been tossed, and end up looking like the picture on the left. We will not pay attention to the positions of the sticks. We will only look at what the faces show. In particular, we will assume for now that no two sticks touch. (In the actual game, the fact that two sticks may touch, or cross, turns out to be very important. We postpone that discussion until later.)

Warm-up puzzle: In how many ways can the sticks land? Draw pictures of each way.
Each different way in which the sticks can land is called an *outcome* of the throw, or of the player’s turn.

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**Family Circle Part 1: The Other Side**

The three ashbii sticks have been thrown, and end up as shown at right. Can you tell which stick is which? That is, can you tell what is on the other side of each stick? Remember: There is only one stick of each type.

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...The power of the world always works in circles, and everything tries to be round. In the old days when we were a strong and happy people, all our power came to us from the sacred hoop of the nation, and so long as the hoop was unbroken the people flourished.

—Black Elk (1863-1950) religious leader (Oglala Sioux), from *Black Elk Speaks* (1961)
How about for these pictures? What is on the other side of each stick?

Family Circle Part 2: Scoring the Game

We consider just part of the game of Ashbii, whose mathematical basis is not at all simple. Part of the scoring scheme involves looking at the outcome of a toss (the appearance of the three sticks).

A face showing black or half-black gives the player one point.
A face showing red or half-red gives the player two points.
Points are not added. If two faces show black or half-black, that is only one point for the player.
An outcome receives only the highest score earned by one face.

For example, the leftmost outcome in the pictures above gets two points, for the half-red face. An outcome with two black faces and a half-black face (not shown) gets one point, just for showing a black face.

So each outcome is worth either one point or two points. (There is more to the scoring of this game, but we will just now talk about this particular part of the game.)

1. A player can get 10 points by throwing seven tosses: 3 two-point tosses and 4 three-point tosses. How else can she get ten points? What are the largest and smallest number of tosses she needs to get 10 points?
2. Same question, but the player has gotten 13 points.
3. Suppose you had postage stamps worth 10 cents and postage stamps worth 20 cents. In how many ways can you put them on a package that costs $1.00 to mail?
4. Which is more difficult: question 1 above or question 3 above?
5. Suppose you had postage stamps worth 10 cents and 30 cents. In how many ways can you put them on a package that costs $1.00 to mail?

Ask Bluebird

QUESTION—What is a cube root? From Ye-Shiao T.

BLUEBIRD SAYS—Well, diced carrots are cubed roots!

Seriously: What are square roots? We can describe the square root of N as a positive number, two copies of which multiply up to N. The square root of 100 is 10 because 10×10 = 100.

So what are cube roots? The cube root of N is a number, three copies of which multiply up to N. The cube root of 1000 is 10 because 10×10×10 = 1000.

We say ‘a number’, but both the square and cube roots of N are unique. We have to say that the square root of a number is a positive number. But we don’t have to say that the cube root of a number is a positive number. Can you see why not?

FUN FACT OF THE FORTNIGHT The number of people who have ever lived on earth, and who have shaken hands an odd number of times in their lives (with another person) is even.